

Financial Development Beyond the Formal Financial Market*

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Abstract

This paper studies the effects of financial development, taking into account both formal and informal financing. Using cross-country firm-level data, we document that informal financing is utilized more by relatively richer countries than poor countries. To account for this empirical pattern, we build a model in which the supply of informal financing increases with financial development, while the demand for informal financing declines with it. The model generates a hump-shaped relationship between the incidence of informal financing and GDP per capita. Our analysis shows that, at the early stage of economic development, the output loss from financial frictions is reinforced by the low supply of informal financing. Informal financing contributes more to the aggregate output of the richest countries than to that of the poorer countries in our sample.

JEL code: E44, O17, O47

Keywords: informal financing, financial development

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1 Introduction

Since [Schumpeter \(1911\)](#), many economists have argued that a well-developed financial market is crucial to promote economic growth. Papers in the financial development literature use a variety of indicators to measure the level of financial market development in different countries and over time. For example, [Greenwood, Sanchez and Wang \(2010\)](#) uses interest rate spreads to measure the effectiveness of the financial market. [Djankov, McLiesh and Shleifer \(2007\)](#) constructs an indicator called “private credit,” which includes loans issued by the commercial banks and other financial institutions to the private sector. [Buera, Kaboski and Shin \(2011\)](#) instead uses “external financing,” which, in addition to “private credit,” also includes funds obtained by the private sector from the bond and equity market.

However, these indicators suffer from one key caveat: they measure formal financing activities in the economy, and exclude financing from lenders that do not specialize in financial intermediation, such as moneylenders, friends, family, and input suppliers. These loans are relationship- and reputation-based, unregulated, and most likely do not appear on a firm’s balance sheet. They are inherently very difficult to measure, especially at the aggregate level. We label them as *informal* financing, in contrast with the *formal* financing provided by financial intermediaries and the financial market.

One might expect that poor countries rely more on informal financing to mitigate the loss from financial frictions. If this is true, the importance of a well-developed formal financial market might be overstated. However, using the World Bank Enterprise Survey (WBES) and China and U.S. manufacturing firm-level data, we document the opposite pattern. In WBES, a dataset consisting mostly of low and middle income countries, informal financing as a share of total financing increases slightly with GDP per capita. In addition, financially constrained firms in richer countries also use relatively more informal financing than financially constrained firms in poorer countries.

We show that this empirical pattern can be generated by a simple model of heterogeneous entrepreneurs facing financial frictions and the coexistence of formal and informal financing. The intuition is simple: Consider an entrepreneur who needs to finance her production, but formal financing is limited by the fundamental contractual enforcement problem in the economy. Potential informal lenders such

as her family and input suppliers have an advantage in lending to her because they have a better enforcement over her repayment of loans. But unlike banks, these informal lenders are themselves faced with financial constraints. A less developed formal financial market and a lower wealth level of potential lenders could both result in a lower supply of informal financing. Therefore, even if entrepreneurs are more financially constrained in a poor country, they use fewer informal loans because their potential informal lenders are too constrained and too poor to lend to them.

In the model economy, there is a continuum of islands, each of which is populated by workers and heterogeneous entrepreneurs with different productivity and wealth. All entrepreneurs have access to an economy-wide formal financial market. The size of formal loans is limited by a collateral constraint, which can be relaxed with the development of the formal financial market and the accumulation of wealth. Entrepreneurs from the same island can also borrow from each other through an informal channel. The informal financing facilitates resources to move to a more productive entrepreneur of the island when she is constrained on the formal financial market. The demand for informal loans declines when the formal market becomes relatively more efficient. The supply of informal loans, however, is determined by the less productive entrepreneur's access to formal loans, which increases with her wealth and the efficiency of the financial market. Therefore, when the supply-side force dominates, the incidence of informal financing could increase with economic development.

Building on a calibrated version of the model, our analysis suggests the informal financing plays a quantitatively more important role in the richest countries of our sample. The use of informal financing accounts for 3.2 percent of GDP of the richest quintile of the countries. In contrast, informal financing contributes to only 2.75 percent and 2.05 percent of the GDP of the 1st and 2nd poorest quintile of countries, respectively. In short, at the early stage of economic development, the output loss from financial frictions is amplified by the existence of informal financing.

Lastly, our model predicts that the output gain from informal financing would eventually decline. This happens when an increasingly larger share of firms' need for external financing could be met by the formal financial market. The hump-shaped pattern of the size of informal financing is also confirmed using the aggregate data of a sample of OECD countries.

Literature review This paper belongs to the following strands of literature. First, it contributes to the empirical literature that studies informal financing and firm performance. This strand of literature often takes firm-level data from a specific country and studies the role of informal financing for firms with limited access to formal financing. The results are rather inconclusive. Take the studies on informal financing in China as an example: while [Allen, Qian and Qian \(2005\)](#) shows that informal financing is important to promote growth in China, [Ayyagari, Demirgüç-Kunt and Maksimovic \(2010\)](#) finds that firms with access to formal credit (bank loans) grow faster than firms that utilize only informal financing. [Degryse, Lu and Ongena \(2013\)](#) instead shows that informal financing that is simultaneously granted with formal financing contributed to firm growth. This paper contributes to the literature by focusing on cross-country study of informal financing and emphasizing the relationship between informal financing, formal financing, and economic development.¹

Second, this paper is one of a few that model explicitly the interaction between formal and informal financing (see [Karaivanov and Kessler, 2018](#) and [Madestam, 2014](#)). Similar to this paper, [Madestam \(2014\)](#) provides a model of informal financing and generates the substitution between informal and formal financing in equilibrium.² This paper differs from [Madestam \(2014\)](#) in two dimensions. First of all, in [Madestam \(2014\)](#) the degree of substitutability between the two types of financing is determined by the monopolistic power of the formal lenders, while in this paper, it is determined by the informal lenders' access to formal financing. This difference allows us to link the substitutability with the level of economic development. Secondly, this paper builds informal financing into a quantitative framework to examine the aggregate effect of informal financing.

The third strand of literature this paper belongs to is that which quantifies the impact of financial friction on aggregate productivity loss. It is widely understood that resource misallocation across firms can account for a large fraction of productivity differences across countries ([Hsieh and Klenow, 2009](#), [Guner, Ventura and Xu, 2008](#) and [Restuccia and Rogerson, 2008](#)). Many papers have since shown that finan-

¹[Allen, Qian and Xie \(2018\)](#) exploits cross-region differences in China and documents that certain type of informal financing is also more prevalent in regions with better access to formal financing.

²Both papers borrow insights from the literature on trade credit (see [Biais and Gollier, 1997](#) and [Burkart and Ellingsen, 2004](#)) that the existence of informal financing reflects a certain comparative advantage of informal lenders in extending loans to borrowers.

cial friction leads to resource misallocation and quantified the aggregate productivity loss from financial frictions (see [Buera, Kaboski and Shin, 2011](#), [Greenwood, Sanchez and Wang, 2010](#), [Midrigan and Xu, 2014](#) and [Moll, 2014](#)). Our paper expands this literature by incorporating informal financing into the framework and quantifying its importance. [Jones \(2013\)](#) points out that the loss from misallocation can be amplified by the misallocation of input goods. The evidence in our paper suggests that trade credit—the informal and implicit loan from input suppliers—might be crucial in understanding why there is less misallocation in the U.S. than in China.

2 Empirical evidence

In this section, we combine several datasets to document empirical patterns of the cross-country differences in informal financing at the country level (section 2.2) and at the firm level (section 2.3).

2.1 Data and sample selection

Penn World Table From the Penn World Table version 8.0, we take the data of real GDP (rgdpe) and population (pop) to compute the real GDP per capita. The logarithm of real GDP per capita is computed as the logarithm of average GDP per capita over the period 2000–10 for each country. A summary of real GDP per capita at country level can be found in column (2) of Table A1.

Financial Development and Structure Dataset We use this dataset to compute the ratio of external financing to GDP.³ Similar to [Buera, Kaboski and Shin \(2011\)](#), this ratio is computed as the sum of 1) private credit by deposit money banks and other financial institutions as a percent of GDP (pcrdbofgdp), 2) stock market capitalization as a percent of GDP (stmktcap) multiplied by 0.33 (average book-to-market ratio in the U.S.), and 3) private bond market capitalization as a percent of GDP

³A detailed discussion of this dataset can be found in [Čihák et al. \(2012\)](#).

(prbond). A summary of the indicator across different countries can be found in column (3) of Table A1.

World Bank Enterprise Survey We use the World Bank Enterprise Survey (WBES) standardized data (2006–14) to document informal versus formal financing across countries.⁴ There are 109 countries in this dataset. On average, each country was surveyed for two years. We first compute the ratio of informal to formal financing for firms in this dataset; we then use this firm-level ratio and sample weights provided by WBES to compute the country-level average.

To compute the share of fixed asset investment financed by informal loans, we calculate the sum of variable k5f (purchase on credit from suppliers and advances from customers) and k5hd (loans from moneylenders, friends, relatives, etc.). Similarly, to compute the share of working capital financed by informal loans, we calculate the sum of variable k3f (purchases on credit from suppliers and advances from customers) and k3hd (loans from moneylenders, friends, relatives, etc.).⁵ The average informal financing as a share of total investment and working capital is presented in columns (5) and (6) in Table A1 for all countries. Summary statistics of the firm-level variables of this dataset can be found in Table A2.

The World Bank also publishes country-level financial indicators that they calculate using the World Bank Enterprise Survey. We take from this dataset the share of fixed assets investment and working capital financed by supplier credit.⁶ We use these indicators as a robustness check of our country-level indicators generated from firm-level data. and to discipline our quantitative model.

OECD Statistics We use the OECD Statistics (from Haver) of the nonfinancial corporate sector to compute the ratio of trade credit to gross value added from 2000 to

⁴The World Bank Enterprise Survey has been used to study informal financing in China (see Ayyagari, Demirgüç-Kunt and Maksimovic, 2010). It is also used to study cross-country income differences and to discipline quantitative models (see Ranasinghe and Restuccia, 2018).

⁵We drop all establishment-level observations and all observations with missing value. And we include the survey (identified by country-year) only if it contains more than 100 observations. Observations from Kosovo and West Bank And Gaza. Observations from Cambodia are also excluded because information on firm size and sector is missing.

⁶There are two reasons why these country-level indicators are different from the ones we constructed from the Enterprise Survey firm-level data. First, they use different years of the Enterprise Survey sample. Second, they consider only supplier credit, which is part of informal financing according to our definition.

2011. Data are available for the following OECD countries: Austria, Belgium, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Israel, Italy, Korea, Latvia, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovenia, Spain, Sweden, Turkey, UK, and US.

Annual Survey of Chinese Manufacturing Firms We use the Annual Survey of Chinese Manufacturing Firms (2005–07) to study trade credit of Chinese firms. These data cover the universe of manufacturing firms with an annual gross revenue of five million RMB or more. Although the survey covers a longer period of time, we take only the years 2005–07, in which trade credit information is available. Summary statistics of firms in this dataset can be found in Table A3.⁷

Survey of Small Business Finance and Compustat We use these two datasets to study trade credit of the U.S. firms. The Survey of Small Business Finance is available only for the fiscal years of 1987, 1993, 1998, and 2003. We complement this dataset with the Compustat NA annual dataset for the fiscal years 1987, 1993, 1998, and 2003. Unlike the Chinese data, the U.S. sample is a less representative sample. But it has the advantage of covering both the very small and very large firms in the economy. Summary statistics of the firms can be found in Table A3.⁸

2.2 Aggregate-level pattern

As discussed in the introduction, there exists a strong positive correlation between the income level of an economy and the measured level of formal financial market development.⁹ Conventional wisdom says that poor countries might use relatively more informal financing than rich countries, i.e. a substitution of informal for for-

⁷We drop all foreign firms in the sample. We drop all observations with missing information on the firm type, age, and sector. Following the literature, we winsorize the top and bottom 5th percentile of the distribution in the ratio of accounts receivable to sales and the ratio of accounts payable to sales, respectively (see Kim and Shin, 2012).

⁸We keep only the manufacturing firms to be comparable with the Chinese firm-level data and drop the observations with missing information. Similarly, we winsorize the top and bottom 5th percentile of the trade credit distribution.

⁹Figure A1 displays the positive correlation between real GDP per capita (averaged over 2000–11) and the level of formal financial market development, as measured by the ratio of external financing to GDP (averaged over 2000–11) in a sample of 136 countries.

mal financing, because they are more constrained on the formal financial market. However, as shown in Figure 1, in this sample of 109 countries in the WBES, the share of informal financing in total fixed assets investment and working capital in fact increases slightly with the income level of the countries.¹⁰

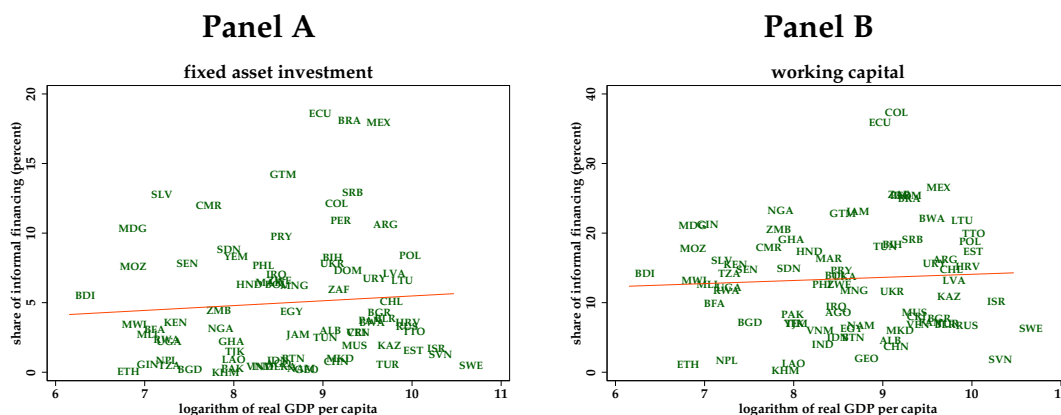


Figure 1: Share of informal financing

Notes: This figure shows the correlation between the logarithm of real GDP per capita (x axis) and the share of informal financing (y axis) in fixed assets investment (Panel A) and in working capital (Panel B). Data for informal financing are calculated using the World Bank Enterprise Survey, and real GDP per capita is calculated using the Penn World Table.

The WBES is the only cross-country firm-level dataset with information about informal financing and a good coverage of low income countries. The caveat of this dataset is its relatively small sample size, which might have contributed to the large variance in informal financing across countries, as shown in Figure 1. To examine further the correlation between informal financing and the income level of countries, we perform the following two additional exercises.

First, in Figure A3, we restrict the WBES sample to countries with more than 500 firm-level observations. In this restricted sample, the positive correlation between informal financing and the income level of the countries becomes stronger and clearer for fixed asset investment (Panel A) and remains unchanged for working capital financing (Panel B).

Second, in Figure 2, we plot the ratio of trade credit to gross value added and the

¹⁰ In Figure 1, we calculate the share of informal financing at the country-level as the weighted average of all firms in each country. As a comparison, in Figure A2 in the Appendix, we use the country-level indicators of the share of fixed asset investment and working capital financed by supplier/trade credit. The patterns in Figure 1 and Figure A2 look very similar.

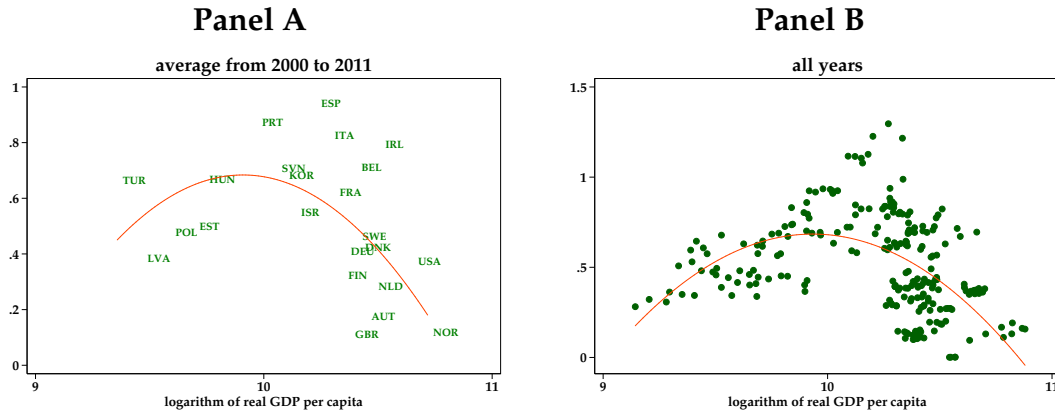


Figure 2: Ratio of trade credit to gross value added

Notes: This figure shows the correlation between the logarithm of real GDP per capita (x axis) and the ratio of trade credit to gross value added of the nonfinancial corporate sector (y axis). Data for the ratio of trade credit to gross value added are taken from the OECD Statistics and real GDP per capita is calculated using the Penn World Table. Panel A plots the average value for each country over the period 2000-2011, while Panel B plots all countries and all years. Greece and Luxembourg are excluded. The ratio of trade credit to gross value added in Greece is 0.04, significantly lower than the rest of the sample. Luxembourg is excluded because of its status as an off-shore tax haven.

real GDP per capita for a sample of OECD countries. The countries in this sample are much more developed than ones in the WBES dataset. Turkey (TUR), one of the poorest OECD countries, is among the richest ones in the WBES sample. Interestingly, Figure 2 shows a clear hump-shaped pattern. The size of trade credit first increases with income, peaks at approximately the income level of Korea (KOR) and Israel (ISR), and declines after that. The countries to the right of the peak are largely missing in Figure 1, because WBES under-samples the most developed countries. This could explain why Figure 1 misses the decreasing part of the curve. Since financial frictions play a more important role in low and middle income countries, in the rest of the paper, we restrict our analysis to the sample of countries in WBES.

2.3 Firm-level pattern

In this section, we study the substitutability of informal financing for formal financing in different countries at the firm level.

World Bank Enterprise Survey For each country c in the WBES, we pool the surveys from different years, and run the following regression:

$$inf_{ist} = \alpha + \beta_c I_constrained_i + \chi_{st} + I_young_i \times I_small_i + \gamma_i + \varepsilon_{ist}. \quad (1)$$

In the regression,

- inf_{ist} is the percent of fixed assets investment (working capital) of firm i in sector s of year t that is financed through informal channels.
- $I_constrained$ is a dummy indicator of whether the firm i is financially constrained. A firm is defined as being financially constrained if it reports that access to finance is its biggest obstacle of growth.
- χ_{st} is a set of sector \times year fixed effects.
- I_young_i is a dummy indicator of whether the firm is young (≤ 5 years old).
- I_small_i is a dummy indicator of whether the firm is small (≤ 10 employees).
- γ_i is a dummy indicator of firm i 's type: whether it is government-owned, private, or foreign.

The estimated coefficient $\hat{\beta}_c$ is the object of interest. In country c , compared with financially unconstrained firms, $\hat{\beta}_c$ percent more fixed asset investment (working capital) of the constrained firms is financed through informal channels. We expect $\hat{\beta}_c$ to be positive, meaning constrained firms borrow relatively more through informal channels compared with unconstrained firms.

In Figure 3, we plot the estimated coefficient $\hat{\beta}_c$ against the GDP per capita of country c . In Panel A, the dependent variable is the percent of informal financing in fixed assets investment, and in Panel B, it is the percent of informal financing in working capital. In both cases, we see that for almost all countries, $\hat{\beta}_c$ is positive. What is more interesting is that in both cases, $\hat{\beta}_c$ increases with the income level of the country. In other words, financially constrained firms in developed countries rely more on informal channels to finance their production than do their financially constrained counterparts in developing countries.¹¹

¹¹The pattern holds if we run a regression by pooling the observations from all the countries. As shown in Table A4 in the Appendix, the interaction term between the indicator of being constrained

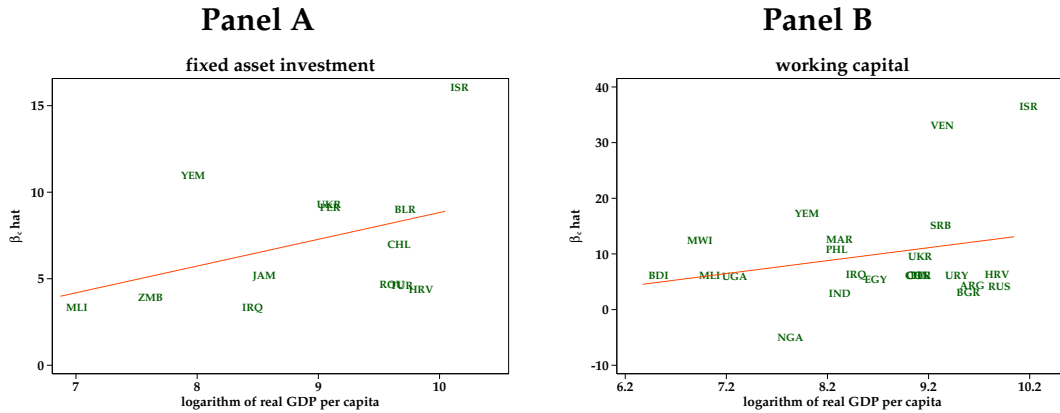


Figure 3: Substitutability of informal to formal financing increases with income

Notes: This figure shows the correlation between the logarithm of real GDP per capita (x axis) and the estimated coefficient $\hat{\beta}_c$ (y axis) (see regression equation 1). Each point in the figure represents one country. The figure plots only the countries whose estimated $\hat{\beta}_c$ is significant at the 5 percent level. The positive correlation between $\hat{\beta}_c$ and income level is also valid in the whole sample (see Figure A4).

China and U.S. manufacturing firms The WBES dataset suffers from several important limitations. Since the data is designed to study economic development issues, it under-samples the most developed countries, such as the United States. We therefore look into firms in China and the U.S. to confirm that the same pattern can be found in more developed countries.

In this section, we focus our analysis on 1) one type of informal financing—trade credit, and 2) firms in two countries—China and the United States. We use 1) the Annual Survey of Chinese Manufacturing Firms (2005–07) to study the Chinese firms and 2) a pooled sample of the Compustat and Survey of Small Business Finance (SSBF) to study firms in the United States.

For the U.S. and China, we run a regression of the following form,

$$y_{ist} = \alpha + \beta_{c,1}I_{p50} + \beta_{c,2}I_{p75} + \beta_{c,3}I_{p100} + \chi_{st} + \gamma_i + \varepsilon_{ist},$$

in which $c \in \{China, U.S.\}$ denotes the two countries.

and log GDP per capita is positive and significant in column 1,3, and 4. In addition, column 5-8 of the table show that an interaction term between the financial constraint indicator and log external financing to GDP is also positive and significant, which suggests that the substitutability of informal to formal financing increases with the development of external financial market.

Table 1: Trade credit and firm size: U.S. versus China

	(1)	(2)	(3)	(4)	(5)	(6)
25th to 50th percentile	2.398*** (0.198)	-7.439*** (0.330)	9.837*** (0.366)	3.267*** (0.0478)	2.315*** (0.0432)	0.952*** (0.0493)
50th to 75th percentile	2.784*** (0.205)	-11.45*** (0.341)	14.23*** (0.378)	4.585*** (0.0484)	3.798*** (0.0438)	0.786*** (0.0499)
75th to 100th percentile	2.520*** (0.210)	-12.55*** (0.350)	15.07*** (0.389)	5.086*** (0.0508)	4.916*** (0.0460)	0.170*** (0.0524)
Dependent variable	AR/S	AP/S	Net AR/S	AR/S	AP/S	Net AR/S
Country	U.S.	U.S.	U.S.	China	China	China
N	15317	15317	15317	705312	705312	705312
AR2	0.195	0.165	0.210	0.113	0.0773	0.0245

Notes: The dependent variable for the regressions are the ratio of accounts receivable to sales in column (1) and (4), the ratio of accounts payable to sales in column (2) and (5), and the ratio of net accounts receivable to sales in column (3) and (6). Column (1)-(3) use data for the U.S. firms and column (4)-(6) use data for the Chinese firms. All regressions include a set of sector times year fixed effects and a set of dummies of firm types. Both the U.S. and the Chinese datasets only contain manufacturing firms. The Chinese dataset contains both state-owned enterprises (SOE) and private enterprises.

The dependent variable of this regression is the ratio of accounts receivable to sales, the ratio of accounts payable to sales, and the ratio of net accounts receivable to sales of firm i in sector s and year t . We have three dummy variables, I_{p50} , I_{p75} , and I_{p100} , indicating, in terms of total asset size, whether the firm belongs to the 25th to 50th percentile, 50th to 75th percentile, or 75th to 100th percentile. The control group in this regression is firms that belong to the bottom 25 percentile in terms of total assets, i.e. the smallest firms. Other control variables include a set of sector-year fixed effects χ_{st} , and a set of dummy variables γ_i that controls for firm types.¹²

The objects of interest are the estimated coefficients $\hat{\beta}_{c,1}$, $\hat{\beta}_{c,2}$, and $\hat{\beta}_{c,3}$. Since many empirical papers suggest that small firms are on average more financially constrained than large firms, if trade credit can substitute for the lack of access to formal financing, we should see that larger firms borrow significantly less trade credit and/or lend significantly more trade credit.

As shown in Table 1, this is indeed the case for the U.S. firms. Larger firms in the U.S. lend significantly more trade credit (column 1) and borrow significantly less

¹²In the U.S. data, we distinguish between the following firm types: Compustat firm or SSBF firm; and corporate or non-corporate firm. In the Chinese data, we control for the following firm types: State-owned, private, and collectively owned.

(column 2). Not surprisingly, in net terms, large firms lend significantly more than their smaller counterparts (column 3). However, this pattern does not hold for the Chinese firms. As shown in column (4), smaller firms do borrow slightly more trade credit; however, they also seem to lend slightly more to their customers (column 5). In net terms (column 6), the difference in trade credit lending among different-sized firms are very small (all estimates are lower than 1 percentage points). The difference between smallest and largest firms is only 0.17 percentage point.¹³

2.4 Discussion

Taking stock, this section documents the following three facts about informal financing. First, at the firm level, there is a certain degree of substitution between informal and formal financing. Financially constrained firms use more informal financing compared with unconstrained firms (see Table 1 and Figure 3). Second, at the country level, it seems that informal and formal financing are complements: as the income of a country increases, both formal and informal financing increase (see Figure A1 and Figure 1). Lastly, the substitutability between informal and formal financing at the firm level increases with economic development (see Figure 3). These three facts motivate the model in the following section.

It is also interesting to note that our results still hold when only considering one type of informal financing—trade/supplier credit. Allen, Qian and Xie (2018) emphasizes the differences between “constructive” informal financing including family loans and trade credit, and “underground” financing, such as moneylenders. In reality, there is perhaps a pecking order among the different types of informal financing for firms, such that they would exhaust the “constructive” loans before borrowing from the “underground” channel. In this paper, however, instead of looking into the differences, we emphasize two characteristics shared by all informal financing. First, the lenders are not specialized financial intermediaries. They do not have access to a deep pocket of funding. Second, they are excluded in the measures of the (formal) financial development as discussed in the introduction. Therefore, condi-

¹³We also run the regressions excluding the state-owned enterprises (SOEs) in the Chinese sample and include all non-financial firms in the U.S. sample. The results are very similar (see Table A5). The results are also very similar if we use only the Compustat sample for the regression of the U.S. firms (see Shao, 2017 for details).

tional on the availability of data, we do not distinguish between the different types of informal financing in our analysis.

3 Model

This section introduces a dynamic general equilibrium model with heterogeneous entrepreneurs faced with frictional formal and informal financing.

3.1 Economic environment

Time is discrete, with an infinite horizon. There is one good in the economy, which is used for consumption and investment.

There is a continuum of islands, each of which is populated by one household with two entrepreneurs and another household with N workers. The entrepreneurs use labor and capital to produce goods. The workers provide labor inelastically to the market and earn wages for their work. Unlike the entrepreneur households, the worker households do not have access to the capital market, i.e. they are “hand-to-mouth.”

3.2 Preference, endowment, and production technology

The entrepreneurs operate a decreasing return to scale production technology that transforms capital and labor into the consumption/investment good, such that

$$y_t = Az_t k_t^\alpha l_t^\chi,$$

where A is the economy-wide total factor productivity (TFP) and z_t is the idiosyncratic productivity shock faced by the entrepreneur, which follows an exogenous stochastic process.

For a worker household, the preference of its n^{th} member is time-separable with an instantaneous utility function of the CRRA form $u(c_{n,t}) = \frac{c_{n,t}^{1-\sigma} - 1}{1-\sigma}$. The utility of

the worker household over a sequence of consumption $c_n = \{c_{n,t}\}_{t=0}^{\infty}$ is

$$U^w(c_1, \dots, c_N) = \sum_{t=0}^{\infty} \beta^t \sum_{n=1}^N u(c_{n,t}),$$

which means that the household puts the same weight on the welfare of its members.¹⁴

Similarly, for an entrepreneur household, the preference of the m^{th} member is time-separable with an instantaneous utility function of the CRRA form $u(c_{m,t}) = \frac{c_{m,t}^{1-\sigma} - 1}{1-\sigma}$. The utility of the entrepreneur household over a sequence of consumption $c_m = \{c_{m,t}\}_{t=0}^{\infty}$ is

$$U^e(c_1, c_2) = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \sum_{m=1,2} u(c_{m,t}).$$

The expectation is taken over a stochastic stream of consumption $\{c_{m,t}\}_{t=0}^{\infty}$ and idiosyncratic productivity $\{z_{m,t}\}_{t=0}^{\infty}$.

3.3 Timing

At the beginning of period t , the entrepreneur households enter each period with wealth a_t , distribute the wealth to the two entrepreneurs in the household ($a_{1,t} + a_{2,t} = a_t$) and send them out to produce. At the same time, the worker households send their members out to work. After the entrepreneurs' idiosyncratic productivity $z_{1,t}$ and $z_{2,t}$ are realized, they seek financing by going to the formal financial market to take out formal loans and, if the formal loans are insufficient, they search for the other entrepreneur from the same household to borrow from her informally. With probability $\epsilon \in [0, 1]$ the search is successful. Then production begins. At the end of production, the entrepreneur and workers return to the households with their production profit and wage income, respectively. The households then choose consumption and saving into the next period a_{t+1} . An illustration of the timing can be found in Figure 4.

¹⁴The workers' wage is deterministic, therefore there is no expectation operator over the future utilities.

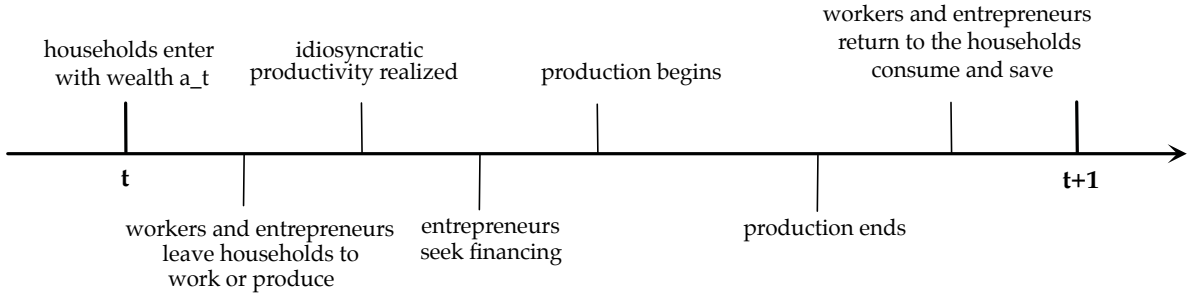


Figure 4: **Timing**

3.4 Markets and frictions

The workers in the economy are perfectly mobile across islands. There exists an economy-wide competitive labor market with wage w that clears the market.

There is an economy-wide competitive formal financial market. Following the literature, we model the formal financial market as a capital rental market, from which the entrepreneur households from all islands can save and borrow at a risk-free interest rate r .

The financial frictions in the economy originate from the limited enforcement over the repayment of formal loans. As a result, the entrepreneurs' borrowing from the formal financial market is limited by the amount of collateral they own. The "no default" formal loan contract requires that $\zeta k \leq a$, where $\zeta < 1$ is the share of capital that entrepreneurs can run away with if they default on the contract. The size of formal loan is therefore constrained, such that $k \leq \gamma a$, where $\gamma = \frac{1}{\zeta}$.

Besides accessing the economy-wide formal financial market, entrepreneurs from the same island could also borrow from and lend to each other. This within-island lending aims at capturing the informal financing activities in reality. The underlying assumption is that the repayment of informal loans between members in the same island can be perfectly enforced. This comparative advantage gives rise to informal financing within an island. But lenders of informal financing are not a specialized financial intermediary, therefore, they do not have access to a "deep pocket" of funds and are subject to the same constraint on the formal financial market as a borrower is. More formally, and without loss of generality, let i be the potential lender of informal loan, $-i$ the potential borrower, and $\hat{k} \geq 0$ the size of the informal loan.

The formal financial constraints limit the size of capital inputs and informal loan. That is, the following constraints need to be satisfied: $k_i + \hat{k} \leq \gamma a_i$ and $k_{-i} \leq \gamma a_{-i}$.

In addition, one might wonder whether the assumption of a perfect enforcement of the repayment of informal loans is too strong. It is reasonable to assume that both formal and informal lending are affected by the quality of the institutions in the economy. For example, both formal and informal lender could benefit from the protection of a more effective judicial system. To capture this idea, we assume entrepreneurs' search for informal financing is successful only with probability ϵ .¹⁵ The value of ϵ can be country-specific and is calibrated to match the data.¹⁶

The structure of the financial markets in this economy is illustrated in Figure 5.¹⁷

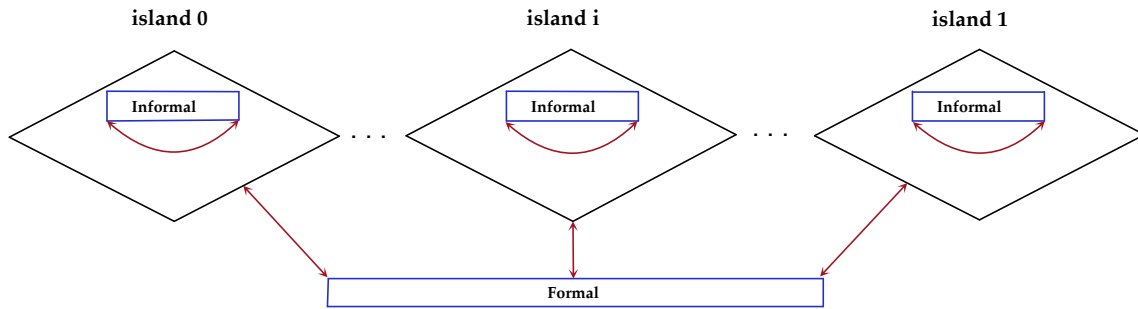


Figure 5: **Financial markets in the economy**

¹⁵Alternatively, we could also assume the enforcement of informal financing is imperfect but better than that of the formal financing.

¹⁶The calibration shows a positive correlation between the formal and informal financial frictions (γ and ϵ). That is, countries with a better formal financial market also faces less frictions in the informal market. However, it is important to note that our key mechanism does not rely on the positive correlation between γ and ϵ . For example, in Figure 6, the model generates a hump-shaped pattern of the incidence of informal financing with the value of ϵ being constant.

¹⁷The model is akin to the island economy in Gertler and Kiyotaki (2010). The informal financial market is analogous to the banks of Gertler and Kiyotaki (2010), and the economy-wide formal financial market is analogous to the inter-bank lending market.

3.5 Discussion

Several assumptions of the model merit discussion. First, we choose decreasing return to scale production function instead of constant return to scale to better match firm heterogeneity in the data. Second, in order to keep the model tractable, we assume that the consumption and saving decisions are made at the household level to rule out multiple equilibria.¹⁸ Third, we abstract from individual occupational choice (entrepreneurs versus workers) because with occupational choice and a decreasing return to scale production technology, the household profit function can be convex-concave under some parameter values. It is well known that a convex-concave profit function could lead to multiple equilibria in the dynamic model (see Skiba, 1978).

4 Recursive competitive equilibrium

This section presents the optimization problem faced by individuals in the economy and defines the recursive competitive equilibrium.

The problem faced by the worker is very simple: the workers provide one unit of labor inelastically to the market and bring back to the household their wage w . Since the worker household is hand-to-mouth, they consume their wage every period, i.e. $c^w = w$. Now consider the two entrepreneurs from the entrepreneur household of island i . Without loss of generality, we label them as i and $-i$ and assume that entrepreneur $-i$ is more productive than entrepreneur i , that is, $z_i < z_{-i}$. Therefore, entrepreneur i is the potential lender of informal financing on the island and entrepreneur $-i$ the potential borrower. Let $\pi(a, z_i, z_{-i}, \omega)$ be the aggregate profit function of the entrepreneur household in island i with wealth a and productivity z_i and z_{-i} . The state variable $\omega \in \{0, 1\}$ is an i.i.d. shock across all islands indicating whether the search for informal financing opportunity is successful.

If the search for informal financing is not successful ($\omega = 0$), the two entrepreneurs maximize their profit subject to a collateral constraint independently. The optimiza-

¹⁸ If entrepreneurs can make saving decisions on their own, there can be multiple equilibria in the dynamic game between the two entrepreneurs on the same island because the savings of the two entrepreneurs are substitutable to a certain degree.

tion problem of an entrepreneur with productivity \tilde{z} and wealth \tilde{a} reads

$$\tilde{\pi}(\tilde{z}, \tilde{a}) = \max_{k,l} A\tilde{z}k^\alpha l^\chi - (r + \delta)k - wl, \text{ s.t. } k \leq \gamma\tilde{a}. \quad (2)$$

In this case, the total profit of production of the entrepreneur household is the sum of the profit of its two members: $\pi(a, z_i, z_{-i}, 0) = \tilde{\pi}(z_i, \frac{a}{2}) + \tilde{\pi}(z_{-i}, \frac{a}{2})$.¹⁹ The aggregate profit function $\pi(a, z_i, z_{-i}, 0)$ can be solved analytically and is shown to be concave in household wealth a (see the details in Appendix C.1).

On the other hand, consider the case where the search for informal financing is successful. Assume that the lender can make a take-it-or-leave-it offer to the borrower.²⁰ The optimization problem is equivalent to the lender maximizing the total profit of the two entrepreneurs subject to the formal financial constraints, such that

$$\begin{aligned} \pi(a, z_i, z_{-i}, 1) &= \max Az_i k_i^\alpha l_i^\chi + Az_{-i} (k_{-i} + \hat{k})^\alpha l_{-i}^\chi \\ &\quad - (r + \delta)(k_i + k_{-i} + \hat{k}) - w(l_i + l_{-i}), \\ \text{s.t.} \quad k_i + \hat{k} &\leq \gamma \frac{a}{2}, \quad k_{-i} \leq \gamma \frac{a}{2}, \\ k_i &\geq 0, \quad k_{-i} + \hat{k} \geq 0. \end{aligned} \quad (3)$$

where \hat{k} is the size of informal financing. The profit function $\pi(a, z_i, z_{-i}, 1)$ can also be characterized analytically, and it is concave in household wealth a (see details in Appendix C.2).

Definition 1 *The recursive competitive equilibrium consists of prices (r, w) , value function of the entrepreneur household $V^e(a, z_i, z_{-i}, \omega)$, policy functions of the entrepreneur household: consumption $c^e(a, z_i, z_{-i}, \omega)$, inputs $k_i(a, z_i, z_{-i}, \omega)$, $k_{-i}(a, z_i, z_{-i}, \omega)$, $\hat{k}(a, z_i, z_{-i}, \omega)$, $l_i(a, z_i, z_{-i}, \omega)$, $l_{-i}(a, z_i, z_{-i}, \omega)$, and next period wealth $a'(a, z_i, z_{-i}, \omega)$, the consumption of workers c^w , and the stationary distribution of the entrepreneur households $\Omega(a, z_i, z_{-i}, \omega)$, such that*

1. Given the prices, the policy functions of the entrepreneur household solve the production optimization problems 2 and 3, and

¹⁹Notice that since the division of wealth within the household happens before the realization of idiosyncratic productivity and the realization of the idiosyncratic shock is observable only to the entrepreneurs, household wealth a will be divided equally between the two entrepreneurs.

²⁰The bargaining power between the lender and the borrower of informal financing does not affect the final result because the consumption and saving decisions are made at the household level.

- Given the prices, the value function and policy functions of the entrepreneur household solve the following problem,

$$\begin{aligned}
V^e(a, z_i, z_{-i}, \omega) &= \max_{c_i, c_{-i}, a'} u(c_i) + u(c_{-i}) + \beta \mathbb{E}_{z'_i, z'_{-i}} V^e(a', z'_i, z'_{-i}, \omega'), \\
s.t. \quad &c_i + c_{-i} + a' = \pi(a, z_i, z_{-i}, \omega) + (1+r)a, a' \geq 0,
\end{aligned}$$

where the household profit function $\pi(a, z_i, z_{-i}, \omega)$ is characterized in Appendix C.1 and C.2.

- The workers' consumption satisfies their budget constraint, that is, $c^w = w$.
- Interest rate r clears the formal financial market. Wage w clears the labor market.
- The distribution Ω is stationary, such that

$$\Omega(a', z'_i, z'_{-i}, \omega') = \int_{a, z_i, z_{-i}, \omega} \mathbb{I}_{a'=a'(a, z_i, z_{-i}, \omega)} \Upsilon(z'_i, z'_{-i}, \omega' | z_i, z_{-i}, \omega) d\Omega(a, z_i, z_{-i}, \omega),$$

where $\mathbb{I}_{a'=a'(a, z_i, z_{-i}, \omega)}$ is an indicator function and $\Upsilon(z'_i, z'_{-i}, \omega' | z_i, z_{-i}, \omega)$ is the transition matrix of the exogenous state variables.

5 Quantitative analysis

In this section, we calibrate the model (section 5.1) and use the calibrated model for two quantitative analyses. In section 5.2, we study the aggregate effects of the development of the formal financial market, that is, a relaxation in the formal collateral constraint γ . In section 5.3, we compare the gain in aggregate output from informal financing for countries at different stages of economic development.

5.1 Calibration

We divide the countries in our sample into five equal-sized groups by income level. Our benchmark calibration aims at matching the data moments of the richest group of countries in this sample.

Table 2: Summary of calibration

Parameter		Value	Target/Source	Data	Model
A	TFP	1	normalized to be 1	–	–
α	capital share in the production function	0.26	capital share of 1/3	–	–
π	Poisson death rate	0.1	Buera, Kaboski and Shin (2011)	–	–
$\alpha + \chi$	scale parameter in production function	0.78	top 5th pct. earning share	0.30	0.35
N	measure of workers	18	share of entrepreneur	10%	10%
δ	capital depreciation rate	0.06	annual depreciation rate	6%	6%
β	discount rate	0.83	annual risk-free interest rate	4%	4%
μ	Pareto tail	3.4	top 10th pct. employment share	69%	67%
γ	collateral value	1.60	ratio of external financing to GDP	0.42	0.42
ϵ	probability of informal financing	0.39	percent of investment financed by informal finance	9.1%	9.1%

Notes: This table is the summary of calibration of the benchmark model to match the richest quintile of the countries. The top 5th percentile earning share and the top 10th percentile employment share are taken from the U.S. manufacturing establishment statistics following Buera, Kaboski and Shin (2011). The ratio of external financing to GDP and the ratio of informal to formal financing in the data are computed as population-weighted average of all countries in the 5th (richest) percentile of our sample.

More formally, we pick the elasticity of inter-temporal substitution σ to be 2. We calibrate β to match the annual risk-free interest rate of 4 percent. The collateral constraint parameter γ is calibrated to match the formal financing to output ratio. The probability ϵ of finding informal financing is calibrated to match the share of informal financing in the data. We model the exogenous process of idiosyncratic productivity as a Poisson death shock with probability π and a redraw of the idiosyncratic productivity from a Pareto distribution with tail parameter μ . Following Buera, Kaboski and Shin (2011), we set the death shock probability $\pi = 0.1$ and calibrate μ to match the top 10th percentile employment share. The scale of the production function $\alpha + \chi$ is calibrated to match the top 5th percentile earnings share. Table 2 shows a summary of the calibration. As shown in the table, the calibration matches all data moments perfectly with two exceptions: the parameter dictating the production scale ($\alpha + \chi$) generates a top 5 percentile earnings share that is slightly higher than the data (0.3 in the data and 0.35 in the model) and the Pareto tail parameter μ generates a top 10 percentile employment share that is slightly lower than the data (69% in the data and 67% in the model).

5.2 The aggregate effect of financial development

In this section, we examine the aggregate effects of formal financial development by varying parameter γ in the calibrated version of the model. The development of

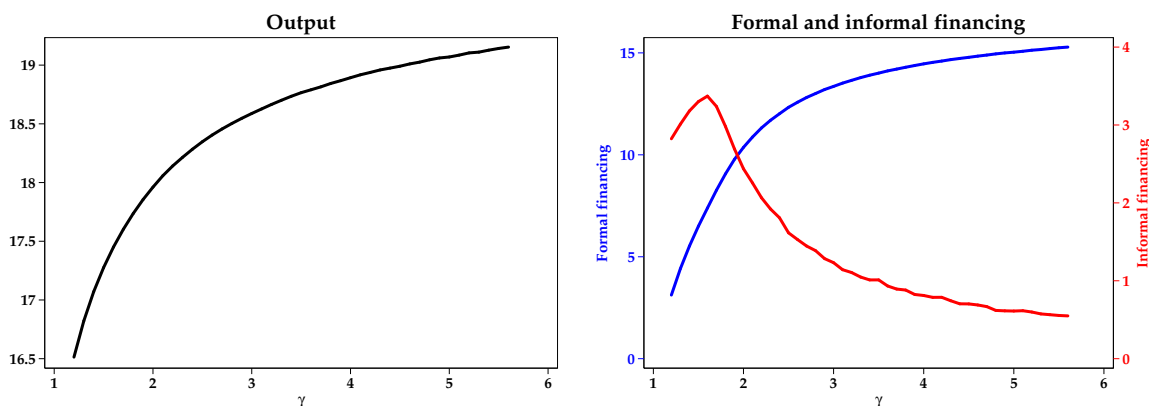


Figure 6: **The aggregate effects of financial development**

the formal financial market can be a result of a better legal institution, technological progress that reduces informal asymmetry, or even urbanization that reduces the transaction cost of banking. The left panel of Figure 6 shows that the aggregate output is increasing and concave in γ . The growth in aggregate output is faster when γ is small and slows down as γ becomes larger. Since the other parameters, such as aggregate TFP, are kept constant, the increase in aggregate output comes solely from a better allocation of resources across heterogeneous entrepreneurs in the economy. The slowdown in the growth of output results from the assumption of decreasing return to scale production technology. Under this assumption, eventually all entrepreneurs become unconstrained when the financial market is sufficiently developed. That is, the economy converges to a frictionless neoclassical economy when γ approaches infinity.

As shown in the right panel of Figure 6, the dynamics of informal and formal financing are perhaps more interesting. The aggregate volume of formal financing follows a similar pattern as that of the aggregate output. However, the aggregate volume of informal financing first increases with γ , peaks at $\gamma = 1.5$, then gradually declines.

Where does the non-monotonicity come from? On the one hand, the supply of informal financing increases with γ . An increase in γ leads to a better allocation of resources and a higher output and wealth. It directly relaxes the constraint on informal financing ($\hat{k} \leq \gamma a$). In addition, the implicit cost of borrowing informal loans, which is equal to the marginal product of capital of the informal lender, is also lower, since she becomes less financially constrained with the development of a formal fi-

financial market.²¹ On the other hand, the demand for informal financing decreases with γ . This is because entrepreneurs exhaust their formal credit before turning to informal loans.²² As the formal financial market develops, more entrepreneurs' financing needs can be met by the formal financial market, therefore the demand for informal loans declines.

In summary, when γ increases, the supply and demand of trade credit move in opposite directions. At the early stage of economic development, the supply force dominates. The aggregate informal financing first increases then declines with γ .²³

5.3 Quantifying the gain from informal financing

Table 3: Calibration of the five quintiles

Quintile	A	Data	Model	γ	Data	Model	ϵ	Data	Model
5	1	N/A	N/A	1.60	0.42	0.42	0.39	9.1%	9.1%
4	0.60	0.50	0.50	1.68	0.45	0.45	0.28	6.5%	6.5%
3	0.39	0.26	0.26	1.37	0.30	0.30	0.21	5.2%	5.2%
2	0.30	0.18	0.18	1.35	0.29	0.29	0.19	4.8%	4.8%
1	0.14	0.06	0.06	1.13	0.14	0.14	0.23	5.5%	5.5%

Notes: This table summarizes the calibration results of the five quintiles of countries in our sample by income (the 5th quintile is the richest and the 1st the poorest). The data moment of output per capita is the average income of all countries in the given quintile. The data moment of the ratio of external financing to GDP and the percent of informal financing in total investment is computed as the population-weighted average of all countries in a given quintile. The aggregate TFP A of the 1st quintile is normalized to be 1. TFP for the other quintiles are calibrated to match the output per capita as a share of richest quintile. The collateral constraint γ is calibrated to match the ratio of external financing to GDP. Friction of informal financing, ϵ , is calibrated to match the share of informal financing in total investment.

In this section, we quantify the gain from informal financing for countries at different stages of economic development. Notice that there are three key parameters

²¹In other words, the interest rate spread between the formal and informal financing decreases.

²²The pecking order is by assumption here. If informal financing incurs a monitoring cost, the pecking order would emerge endogenously in equilibrium because informal financing is more expensive than formal financing.

²³In the exercise, the value of ϵ is fixed. This suggests that the key mechanism of the model does not result from changes in the informal financing frictions.

governing cross-country differences in the model: aggregate TFP A , collateral constraint γ of the formal loan, and the search friction of informal financing ϵ .

We first calibrate our benchmark model to match the five quintiles of countries in our sample. More formally, we calibrate three key parameters—aggregate TFP A , formal financing collateral constraint γ , and informal financing friction ϵ —to match the output per capita, the ratio of external financing to GDP, and the ratio of informal to formal financing respectively for five groups.

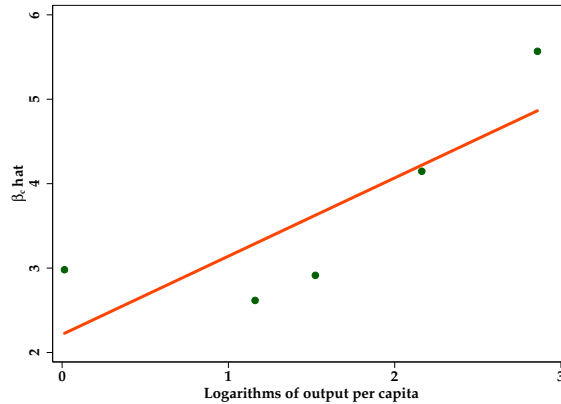


Figure 7: Substitutability of informal to formal financing increases with income

Notes: This figure is the model analog of Figure 3. Each point in the figure represents one quintile in our calibrated model. The y axis is the estimated coefficient $\hat{\beta}_c$ of specification 1 using a model-generated sample of entrepreneurs. The x axis is the output per capita generated by the model.

Table 3 displays the calibration results for the five groups of countries. The calibration matches the aggregate data moments rather well. We also check whether the calibrated model captures the increasing substitutability of informal to formal financing with the increase in income. To this end, we take the sample of entrepreneurs generated by the calibrated model and rerun specification 1. Figure 7 plots the estimated parameter $\hat{\beta}_c$ against the logarithms of the output per capita for the five groups. As is shown in the figure, the estimated $\hat{\beta}_c$ increases with income level, which is consistent with the pattern of the estimated coefficients using real data in Figure 3. However, the slope of the linear fit in Figure 7 (model-generated sample) is slightly lower than the slope of the two linear fit in Figure 3 (data sample). In short, the calibrated model does a decent job in generating the positive correlation between the substitutability of informal to formal financing and the income level of the economy.

With the calibrated model, we proceed to examine the gain from informal financing. For each quintile, we shut down the informal financing channel by setting $\epsilon = 0$ while keeping all the other parameters unchanged. Table 4 shows the output of the benchmark compared with that of the counterfactual economy. With the same technological parameters, the output of the benchmark economy is higher than that of the counterfactual economy for all five groups. The largest gain from informal financing belongs to the richest countries in the sample: the gain in aggregate output is 3.21 percent for the 5th quintile. As a comparison, the 2nd quintile countries benefit the least from informal financing: only 2.05 percent of the aggregate output can be accounted for by informal financing.

Table 4: **Output gain from informal financing by income level**

Quintile	Benchmark	Counterfactual	Percent difference
5	1	0.968	3.21
4	0.499	0.487	2.41
3	0.263	0.257	2.12
2	0.183	0.179	2.05
1	0.058	0.057	2.75

Notes: This table displays the aggregate output of the benchmark and the counterfactual economy by quintile. All the outputs in different quintiles are normalized by the output of the 5th (richest) quintile of the benchmark model.

Intuitively, informal financing helps constrained entrepreneurs expand their production, which contributes to a better allocation of resources and a higher output. As shown in Figure 7, constrained entrepreneurs in the richest countries use the largest amount of informal financing. Not surprisingly, as a result, the richest countries benefit the most from informal financing.

Although it is tempting to conclude that the output gain from informal financing increases with the level of economic development, it is important to remember that developed countries are under-represented in our sample. We conduct the following experiment to study whether the pattern holds when the countries become more developed: we take the richest quintile of countries and allow the financial markets to continue to develop in this economy. More formally, we take the calibration in

Table 2, set $\epsilon = 0.5$, and gradually increase γ .²⁴ The result is presented in Figure A5. The five points on the red line represent the five quintiles of countries in our data, and the blue line is our simulated results. As the formal financial market continues to develop, the gain from informal financing first increases then declines. This non-monotonicity in the gain from informal financing is consistent with the hump-shaped pattern in aggregate informal financing over the development process (Figure 6). It results from the changing supply and demand of informal financing as described in section 5.2.

6 Conclusion

This paper provides a cross-country analysis of informal financing to shed light on its role in the process of economic development. Contrary to traditional views, we find that rich countries—in our sample, they are the middle-income countries—benefit more from informal financing than the poorest countries. More broadly speaking, the goal of this paper is to reach a more comprehensive understanding of financial development and its relationship with economic growth by studying the interactions between different types of financial activities. This paper emphasizes the substitution between informal and formal financing at the firm level and how the substitutability varies with aggregate economic conditions such as TFP and formal financial development. Although the scope of analysis in this paper is limited by data availability, the framework developed in this paper could be easily extended to make use of better data once they become available.

²⁴Essentially, we fix the friction of informal financing and gradually improve the efficiency of the formal financial market. In this experiment, one could also increase both A and γ . In reality, economic development is associated with both technological improvement and financial development. But as pointed out in Greenwood, Sanchez and Wang (2010), for financial development to play a role in the development process, it has to outpace the development of the other sectors. In other words, financial development should be modeled as an increase in γ relative to A rather than an increase in the level of γ only. Here we capture financial development by keeping A constant and increasing γ .

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Appendix

A Summary statistics

Table A1: Income and financial development across countries

Country name	Country code	GDP p.c (US dollars)	Ratio of external financing to GDP (percent)	Share of informal finance in fixed asset investment (percent)	Share of informal finance in working capital (percent)
Angola	AGO	3378.1	8.0	1.2	10.6
Albania	ALB	5729.3	18.5	1.4	5.1
Argentina	ARG	10396.9	32.4	9.5	19.8
Burundi	BDI	513.8	19.8	3.0	10.7
Burkina Faso	BFA	875.6	15.6	3.1	10.1
Bangladesh	BGD	1285.3	34.7	0.3	7.7
Bulgaria	BGR	9276.7	36.1	4.3	7.6
Bosnia and Herzegovina	BIH	7030.4	55.9	8.2	20.8
Belarus	BLR	9804.6	17.6	3.7	7.0
Bolivia	BOL	3250.9	48.4	6.3	11.7
Brazil	BRA	7661.2	70.2	17.8	25.2
Bhutan	BTN	5149.3	25.9	1.4	5.8
Botswana	BWA	8773.2	29.0	4.1	22.8
Chile	CHL	13195.1	147.3	5.6	16.1
China	CHN	5961.1	144.0	0.9	3.8
Cameroon	CMR	1737.4	9.3	11.9	19.1
Colombia	COL	6627.4	38.9	10.0	32.3
Costa Rica	CRI	8523.9	37.9	2.9	8.2
Czech Republic	CZE	19593.6	53.5	3.3	7.2
Dominican Republic	DOM	7118.7	27.8	7.8	25.6
Ecuador	ECU	5709.4	27.5	17.8	32.3
Egypt, Arab Rep.	EGY	4431.1	65.3	4.0	6.8
Estonia	EST	14676.8	76.2	1.6	17.7
Ethiopia	ETH	542.8	20.4	0.1	1.3
Georgia	GEO	4608.9	19.0	1.2	3.6
Ghana	GHA	1816.7	16.3	2.3	19.1
Guinea	GIN	1023.7	4.4	2.5	23.6
Guatemala	GTM	3800.7	23.7	13.7	23.7
Honduras	HND	2936.7	41.4	4.0	13.5
Croatia	HRV	14203.2	63.1	2.9	16.8
Indonesia	IDN	3331.6	32.9	1.1	7.5
India	IND	2630.4	58.5	0.4	5.5
Iraq	IRQ	3848.4	4.2	9.1	11.4
Israel	ISR	24121.6	113.0	1.7	10.2
Jamaica	JAM	4328.7	45.1	2.7	22.8
Kazakhstan	KAZ	8936.6	37.4	2.4	12.6
Kenya	KEN	1191.0	37.2	3.7	17.2
Cambodia	KHM	1524.3	13.4	0.0	0.4
Lao PDR	LAO	2026.9	8.5	0.9	1.5
Sri Lanka	LKA	3647.0	33.2	0.4	20.5
Lithuania	LTU	12911.3	36.1	6.2	25.9
Latvia	LVA	11510.6	51.8	7.2	14.0
Morocco	MAR	3041.0	74.1	6.4	17.3
Madagascar	MDG	801.9	9.3	10.2	21.1
Mexico	MEX	11951.0	40.6	15.7	23.7

Country name	Country code	GDP p.c (US dollars)	Ratio of external financing to GDP (percent)	Share of informal finance in fixed asset investment (percent)	Share of informal finance in working capital (percent)
Macedonia, FYR	MKD	7259.3	30.8	0.9	6.5
Mali	MLI	804.2	17.1	2.8	12.5
Mauritius	MUS	8721.1	83.3	2.0	8.6
Malawi	MWI	624.6	11.8	3.3	13.9
Namibia	NAM	4401.6	49.2	0.6	11.3
Nigeria	NGA	1567.1	24.3	2.6	24.3
Nepal	NPL	1055.3	40.3	1.1	2.4
Pakistan	PAK	2240.4	30.6	0.3	8.6
Panama	PAN	11095.2	90.5	4.5	6.6
Peru	PER	6010.9	39.0	10.4	23.6
Philippines	PHL	3121.4	47.0	8.0	14.7
Poland	POL	13227.9	38.6	9.0	19.7
Paraguay	PRY	4090.2	24.0	9.8	14.9
Russian Federation	RUS	11944.2	44.1	3.3	8.8
Rwanda	RWA	807.9	9.6	2.8	12.6
Sudan	SDN	1854.0	6.5	9.6	17.8
Senegal	SEN	1405.6	21.1	7.8	15.5
El Salvador	SLV	431.7	11.1	8.7	18.5
Serbia	SRB	8119.9	37.2	11.4	27.7
Slovenia	SVN	22091.3	72.4	1.3	1.9
Sweden	SWE	31427.6	174.6	0.6	6.5
Swaziland	SWZ	3902.0	19.7	6.0	22.4
Tajikistan	TJK	1954.3	13.2	2.0	10.1
Trinidad and Tobago	TTO	16917.3	57.0	2.8	21.5
Tunisia	TUN	6033.2	66.3	3.5	18.5
Turkey	TUR	10933.2	32.6	1.2	9.3
Tanzania	TZA	997.7	10.7	0.9	14.9
Uganda	UGA	1069.0	10.4	2.2	13.9
Ukraine	UKR	5909.7	43.2	7.7	13.0
Uruguay	URY	9146.0	33.1	6.0	16.5
Venezuela, RB	VEN	9940.3	19.4	2.9	7.1
Vietnam	VNM	2523.5	69.0	0.5	9.1
Yemen, Rep.	YEM	2592.5	5.6	7.9	8.3
South Africa	ZAF	7040.6	212.7	6.0	25.5
Zambia	ZMB	1136.9	9.4	5.2	22.2
Zimbabwe	ZWE	3928.5	62.7	6.6	12.8

Table A2: Summary statistics of WBES data by country

Country name	Country code	N	size			sector			intra			inter			constrained (percent)				
			small (percent)	median (percent)	large (percent)	manufacturing (percent)	service (percent)	others (percent)	min (percent)	max (percent)	mean (percent)	s.d. (percent)	min (percent)	max (percent)		mean (percent)	median (percent)	s.d. (percent)	
Afghanistan	AFG	303	46.2	42.2	11.6	33.3	11.2	55.4	0	50	0.9	0	5.8	0	100	8.5	0	20.1	48.8
Albania	ALB	294	59.2	35.4	5.4	40.8	25.5	33.7	0	70	1.4	0	7.6	0	100	4.1	0	13.8	18.7
Angola	AGO	190	69.5	26.3	4.2	54.7	10.5	34.7	0	50	0.8	0	5.5	0	100	10.4	0	21.2	61.6
Argentina	ARG	965	35.8	40.5	23.7	67.4	16.1	16.6	0	100	8.4	0	23.1	0	100	23.5	10	29.1	37.1
Bangladesh	BGD	996	23.0	30.9	46.1	83.9	2.9	13.2	0	100	0.5	0	10.0	0	100	6.6	0	16.4	28.4
Belarus	BLR	128	37.5	33.6	28.9	42.2	18.0	39.8	0	100	3.1	0	12.8	0	100	6.0	0	16.4	15.6
Bhutan	BTN	177	40.1	41.2	18.6	17.5	4.0	32.2	0	50	1.4	0	6.9	0	55	5.8	0	12.9	21.5
Bolivia	BOL	392	37.0	43.6	19.4	53.1	18.6	28.3	0	100	5.1	0	16.8	0	100	14.0	0	23.4	25.3
Bosnia and Herzegovina	BIH	175	48.6	37.1	14.3	35.4	26.3	38.3	0	100	9.0	0	22.5	0	100	22.7	0	31.0	14.3
Botswana	BWA	253	56.9	28.9	14.2	33.6	28.9	37.5	0	100	3.9	0	14.2	0	100	22.8	10	28.1	37.5
Brazil	BRA	974	34.5	47.3	18.2	74.5	9.5	15.9	0	100	14.6	0	30.3	0	100	22.1	0	31.3	50.9
Bulgaria	BGR	735	35.5	39.0	25.4	49.3	28.2	22.6	0	100	2.7	0	13.2	0	100	5.2	0	15.5	19.2
Burkina Faso	BFA	124	56.5	33.9	9.7	17.7	26.6	55.6	0	100	2.7	0	13.9	0	100	10.2	0	22.4	68.5
Burundi	BDI	192	68.2	28.1	3.6	38.5	17.7	43.8	0	100	1.2	0	8.4	0	70	10.0	0	14.6	43.8
Cameroon	CMR	115	40.9	42.6	16.5	35.7	24.3	40.0	0	100	11.3	0	23.2	0	100	22.9	10	26.0	52.2
Chile	CHL	1084	27.5	45.4	27.2	72.4	15.8	11.8	0	100	8.7	0	24.6	0	100	20.2	0	28.1	20.4
China	CHN	1103	13.2	41.3	45.5	71.6	2.8	25.6	0	100	0.9	0	6.2	0	100	3.2	0	10.4	5.7
Colombia	COL	1057	34.7	38.9	26.4	70.4	16.5	13.2	0	100	9.8	0	25.1	0	100	27.7	20	30.0	21.9
Congo, Dem. Rep.	COD	417	75.8	20.6	3.6	42.4	19.9	37.6	0	100	3.3	0	12.1	0	100	11.3	0	18.6	58.5
Costa Rica	CRI	246	35.4	43.9	20.7	61.8	19.1	19.1	0	100	3.1	0	13.4	0	100	10.9	0	22.3	39.0
Cote d'Ivoire	CIV	149	66.4	18.8	14.8	28.9	16.1	55.0	0	100	5.0	0	17.7	0	100	7.7	0	22.4	71.1
Croatia	HRV	712	43.7	35.4	20.9	50.1	19.9	29.9	0	100	5.5	0	18.6	0	100	18.2	0	29.1	20.8
Czech Republic	CZE	118	52.5	37.3	10.2	43.2	27.5	37.3	0	100	4.2	0	16.4	0	95	8.3	0	19.9	16.9
Dominican Republic	DOM	152	24.3	42.1	33.6	40.1	27.6	32.2	0	100	10.2	0	25.6	0	92	31.6	30	22.7	17.1
Ecuador	ECU	565	33.5	42.7	23.9	49.2	26.2	24.6	0	100	14.3	0	31.4	0	100	25.5	8	32.3	23.4
Egypt	EGY	322	39.1	35.4	25.5	79.2	3.7	17.1	0	100	3.9	0	15.9	0	100	11.0	0	22.2	37.9
El Salvador	SLV	477	28.7	38.8	32.5	57.4	16.8	25.8	0	100	10.8	0	28.3	0	100	17.7	0	26.2	24.9
Estonia	EST	125	54.4	32.8	12.8	32.8	28.0	39.2	0	70	2.8	0	10.9	0	100	20.5	0	28.2	4.8
Ethiopia	ETH	157	51.6	26.8	21.7	41.4	19.1	39.5	0	20	0.1	0	1.6	0	91	2.3	0	10.6	45.9
Georgia	GEO	130	52.3	32.3	15.4	40.8	27.7	31.5	0	100	2.3	0	12.3	0	100	6.7	0	19.2	19.2
Ghana	GHA	562	63.7	27.9	8.4	55.3	20.6	24.0	0	100	3.6	0	12.5	0	100	16.8	10	20.4	63.2
Guatemala	GTM	488	25.2	36.3	38.5	63.7	15.6	20.7	0	100	10.6	0	26.4	0	100	21.4	0	30.2	19.1
Guinea	GIN	122	87.7	8.2	4.1	59.8	18.0	22.1	0	60	2.6	0	11.1	0	100	22.9	17.5	25.2	56.6
Honduras	HND	297	35.4	34.3	30.3	60.6	19.9	19.5	0	100	4.0	0	16.4	0	100	11.6	0	24.7	19.2
India	IND	1660	24.2	51.6	24.2	78.4	8.3	13.4	0	100	0.2	0	3.4	0	100	4.6	0	13.3	12.1
Indonesia	IDN	312	45.5	29.2	25.3	82.7	7.1	10.3	0	70	0.9	0	6.3	0	100	7.0	0	22.0	11.9
Iraq	IRQ	265	72.1	25.7	2.3	64.9	35.1	0.0	0	100	4.9	0	12.2	0	100	10.1	0	18.2	45.7
Israel	ISR	131	34.4	40.5	25.2	50.4	15.3	34.4	0	92	1.5	0	10.7	0	100	8.2	0	23.1	4.6
Jamaica	JAM	110	30.9	42.7	26.4	48.2	21.8	30.0	0	50	2.0	0	8.3	0	100	24.5	20	20.4	35.5
Kazakhstan	KAZ	157	44.6	38.9	16.6	36.9	31.2	31.8	0	70	1.9	0	8.9	0	100	7.9	0	21.8	12.7
Kenya	KEN	571	38.7	34.5	26.8	59.7	17.7	22.6	0	100	3.7	0	14.2	0	100	22.0	15	23.8	33.3
Laos	LAO	197	45.2	37.1	17.8	17.8	6.6	29.9	0	70	2.2	0	10.5	0	90	4.2	0	14.3	13.7
Latvia	LVA	110	60.0	25.5	14.5	40.0	24.5	35.5	0	100	7.8	0	23.8	0	100	17.4	0	29.7	13.6
Lithuania	LTU	111	45.9	36.0	18.0	37.8	27.0	35.1	0	100	4.9	0	17.2	0	100	25.3	10	33.0	13.5
Macedonia	MKD	169	60.9	31.4	7.7	34.9	24.3	40.8	0	100	2.4	0	12.6	0	100	6.1	0	17.0	18.9
Madagascar	MDG	263	39.5	40.3	20.2	47.1	20.9	31.9	0	100	7.8	0	23.5	0	100	15.9	0	27.5	29.3
Malawi	MWI	164	40.2	34.1	25.6	18.1	16.5	23.2	0	100	3.8	0	12.4	0	85	14.6	0	23.1	40.9
Mali	MLI	256	81.3	16.4	2.3	55.9	23.8	20.3	0	100	2.3	0	10.2	0	100	12.4	10	17.0	59.8
Mauritania	MRT	127	64.6	27.6	7.9	26.0	12.6	61.4	0	100	4.4	0	16.9	0	100	15.2	0	22.6	53.5
Mauritius	MUS	185	44.3	37.8	17.8	38.4	28.1	33.5	0	100	2.7	0	14.6	0	100	6.4	0	18.5	41.6
Mexico	MEX	893	35.9	31.9	32.1	77.5	10.2	12.3	0	100	14.8	0	31.0	0	100	23.5	0	32.4	19.9

Country name	Country code	N	small (percent)	size median (percent)	large (percent)	manufacturing (percent)	sector service (percent)	others (percent)	min (percent)	max (percent)	mean (percent)	median (percent)	s.d. (percent)	min (percent)	max (percent)	mean (percent)	median (percent)	s.d. (percent)	constrained (percent)
Mongolia	MNG	188	43.1	44.7	12.2	35.1	33.0	31.9	0	100	2.2	0	12.1	0	100	11.3	0	23.1	24.5
Morocco	MAR	126	24.6	43.7	31.7	50.8	25.4	23.8	0	100	5.4	0	18.4	0	100	15.8	0	24.9	26.2
Mozambique	MOZ	140	60.0	27.9	12.1	73.6	19.3	7.1	0	100	7.6	0	20.3	0	70	18.4	15	19.3	53.6
Myanmar	MMR	201	59.2	25.9	14.9	55.7	12.9	31.3	0	100	0.9	0	9.5	0	100	6.9	0	19.9	21.9
Namibia	NAM	264	74.5	21.1	4.4	29.9	25.9	44.2	0	100	0.9	0	7.9	0	100	12.8	0	23.5	36.1
Nepal	NPL	268	45.9	41.0	13.1	45.9	17.2	36.9	0	70	0.8	0	6.9	0	70	4.2	0	12.8	19.4
Nicaragua	NIC	324	40.7	42.3	17.0	63.3	14.8	21.9	0	100	2.8	0	12.5	0	100	11.8	0	24.0	20.4
Nigeria	NGA	1671	71.5	24.2	4.3	47.8	20.9	31.3	0	100	3.3	0	10.3	0	100	25.0	20	27.2	16.5
Pakistan	PAK	149	25.5	42.3	32.2	78.5	8.1	13.4	0	80	3.0	0	10.8	0	90	6.2	0	15.0	22.1
Panama	PAN	300	50.0	34.0	16.0	44.7	33.0	22.3	0	100	5.3	0	20.2	0	100	7.4	0	18.4	10.7
Paraguay	PRY	510	35.9	43.9	20.2	51.0	22.4	26.7	0	100	10.0	0	25.6	0	100	14.6	0	24.9	25.7
Peru	PER	906	28.6	43.5	27.9	72.3	12.3	15.5	0	100	8.8	0	24.8	0	100	23.7	0	30.0	14.3
Philippines	PHL	382	18.8	47.6	33.5	74.6	9.9	15.4	0	100	7.2	0	22.1	0	100	15.1	0	27.0	13.4
Poland	POL	162	49.4	34.0	16.7	36.4	27.8	35.8	0	100	9.3	0	21.3	0	100	20.2	0	26.8	17.3
Romania	ROU	244	47.5	33.6	18.9	35.2	23.4	41.4	0	100	3.2	0	13.8	0	100	17.4	0	27.2	32.0
Russia	RUS	1313	40.5	41.1	18.4	38.2	8.1	53.6	0	100	3.6	0	14.5	0	100	10.0	0	22.6	26.0
Rwanda	RWA	221	61.1	25.3	13.6	11.3	10.0	34.8	0	100	3.2	0	13.0	0	100	11.7	0	20.1	27.6
Senegal	SEN	217	68.2	24.0	7.8	59.4	15.2	25.3	0	100	6.9	0	22.6	0	100	15.2	4	24.6	47.9
Serbia	SRB	164	53.0	31.1	15.9	31.7	34.1	34.1	0	100	12.6	0	29.0	0	100	24.4	0	34.2	14.6
Slovenia	SVN	179	49.7	30.7	19.6	34.6	30.2	35.2	0	90	2.8	0	13.3	0	100	3.7	0	15.6	26.3
South Africa	ZAF	284	28.2	50.0	21.8	80.6	10.2	9.2	0	100	6.2	0	20.6	0	100	25.3	20	20.7	14.8
South Lanka	LKA	118	40.7	30.5	28.8	64.4	13.6	22.0	0	100	2.6	0	12.2	0	100	13.8	0	24.0	28.0
Sri Lanka	LKA	118	40.7	30.5	28.8	64.4	13.6	22.0	0	100	2.6	0	12.2	0	100	13.8	0	24.0	28.0
Sudan	SDN	300	68.7	26.3	5.0	16.0	33.3	50.7	0	100	6.8	0	15.9	0	100	14.9	0	23.5	35.7
Swaziland	SWZ	103	69.9	20.4	9.7	26.2	34.0	39.8	0	100	6.1	0	20.8	0	100	23.1	15	27.5	36.9
Sweden	SWE	233	43.3	45.5	11.2	68.2	9.9	21.9	0	100	2.4	0	14.4	0	100	9.0	0	20.4	4.7
Tajikistan	TJK	115	47.8	42.6	9.6	34.8	30.4	34.8	0	80	1.4	0	8.4	0	100	8.0	0	21.0	27.0
Tanzania	TZA	369	56.9	33.9	9.2	68.3	11.1	20.6	0	50	0.9	0	5.3	0	100	18.0	10	21.8	51.5
Trinidad & Tobago	TTO	112	44.6	27.7	27.7	25.0	31.3	43.8	0	100	5.9	0	13.4	0	100	20.7	15	18.1	26.8
Tunisia	TUN	245	26.5	40.8	32.7	60.8	7.8	31.4	0	100	2.9	0	12.8	0	100	18.9	10	23.7	21.2
Turkey	TUR	331	39.9	34.4	25.7	81.0	7.6	11.5	0	90	2.1	0	8.6	0	100	9.6	0	19.9	8.2
Uganda	UGA	376	59.0	31.4	9.6	63.8	14.9	21.3	0	70	2.1	0	8.5	0	100	15.5	10	19.4	44.7
Ukraine	UKR	154	35.1	42.9	22.1	86.4	0.0	13.6	0	100	6.7	0	18.9	0	100	13.8	0	23.2	25.3
Uruguay	URY	578	32.9	43.1	24.0	62.3	13.1	24.6	0	100	5.8	0	19.4	0	100	19.7	0	27.8	20.1
Uzbekistan	UZB	109	33.0	33.0	33.9	44.0	22.0	33.9	0	100	0.9	0	9.6	0	25	0.4	0	2.8	9.2
Venezuela	VEN	210	51.0	32.9	16.2	17.6	7.1	10.0	0	100	7.6	0	24.2	0	100	10.3	0	23.8	19.0
Vietnam	VNM	609	17.1	45.0	37.9	76.0	10.0	14.0	0	100	1.6	0	10.4	0	100	11.2	0	22.0	16.7
Yemen	YEM	128	62.5	27.3	10.2	49.2	20.3	30.5	0	100	10.4	0	28.2	0	100	15.5	0	28.7	38.3
Zambia	ZMB	372	48.7	37.9	13.4	65.9	15.6	18.5	0	100	4.2	0	16.2	0	100	15.6	0	23.6	32.8
Zimbabwe	ZWE	111	29.7	42.3	27.9	61.3	13.5	25.2	0	90	5.6	0	16.8	0	100	11.8	0	22.8	69.4
Yemen	YEM	128	62.5	27.3	10.2	49.2	20.3	30.5	0	100	10.4	0	28.2	0	100	15.5	0	28.7	38.3
Zambia	ZMB	372	48.7	37.9	13.4	65.9	15.6	18.5	0	100	4.2	0	16.2	0	100	15.6	0	23.6	32.8
Zimbabwe	ZWE	111	29.7	42.3	27.9	61.3	13.5	25.2	0	90	5.6	0	16.8	0	100	11.8	0	22.8	69.4

Notes: According to the WBES definition, small firms are those that employ more than 5 and fewer than 19 workers, median firms are those that employ more than 19 and fewer than 100 workers, and large firms are those that employ more than 100 workers. $infa$ is the share of informal finance in fixed asset investment, while $infwuc$ is the share of informal finance in working capital.

Table A3: Summary statistics of Chinese and U.S. firm-level data

Variables	China							United States						
	unit	N	min	max	mean	median	sd	unit	N	min	max	mean	median	sd
Accounts receivable	10'000 CNY	705312	0	38056649.0	7429.7	1466.0	76817.3	million USD	15317	0.0	154034.0	248.0	9.0	2655.2
Accounts payable	10'000 CNY	705312	0	22600000.0	8265.6	960.0	108004.9	million USD	15317	0.0	25422.0	119.3	4.2	761.1
Total assets	10'000 CNY	705312	0	564000000.0	84217.5	12357.0	1511524.8	million USD	15317	0.0	448507.0	1403.9	52.0	8930.5
Ratio of accounts receivable to sales		705312	0	54.9	13.3	7.4	15.0		15317	0.0	36.0	15.3	14.8	8.7
Ratio of accounts payable to sales		705312	0	49.7	10.4	4.8	13.3		15317	1.7	88.7	12.3	7.8	14.1

B Additional figures and tables

Table A4: Substitutability of informal to formal financing (pooling all countries)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
financially constrained	-6.244** (2.703)	-7.528 (7.269)	-5.126*** (1.662)	-7.982 (5.529)	-5.892*** (1.881)	-7.009 (4.816)	-4.139*** (1.256)	-3.527 (3.422)
financial constraint X log GDP pc	0.970*** (0.362)	1.300 (0.801)	0.796*** (0.223)	1.293** (0.614)				
log GDP pc	0.891* (0.535)	0.244 (1.141)	0.556 (0.354)	-0.239 (0.848)				
financial constraint X log ext. fin to GDP					2.386*** (0.695)	3.122** (1.409)	1.764*** (0.414)	1.966** (0.937)
log ext. fin to GDP					-0.258 (0.669)	-2.226 (1.728)	-0.159 (0.279)	-1.272 (1.065)
Dependent variable	fixed assets inv.	working capital	fixed assets inv.	working capital	fixed assets inv.	working capital	fixed assets inv.	working capital
sectorXyear FE	N	N	Y	Y	N	N	Y	Y
firm-type FE	N	N	Y	Y	N	N	Y	Y
firm-size&age FE	N	N	Y	Y	N	N	Y	Y
N	30350	30350	30204	30204	29678	29678	29532	29532
AR2	0.00585	0.00423	0.0274	0.0559	0.00324	0.00633	0.0265	0.0563

Notes: The dependent variables for the regressions are the percent of fixed assets investment that is financed through informal channels (columns 1, 3, 5, 7) and the percent of working capital investment that is financed through informal channels (columns 2, 4, 6, 8). Standard errors are clustered at the country- and sector-level.

Table A5: Trade credit and firm size: U.S. (non-financial) versus China (private)

	(1)	(2)	(3)	(4)	(5)	(6)
25th to 50th percentile	5.711*** (0.177)	-2.113*** (0.320)	7.823*** (0.336)	3.551*** (0.0476)	2.521*** (0.0429)	1.030*** (0.0495)
50th to 75th percentile	7.508*** (0.216)	-10.76*** (0.390)	18.27*** (0.409)	4.897*** (0.0484)	4.014*** (0.0436)	0.883*** (0.0503)
75th to 100th percentile	6.634*** (0.224)	-13.97*** (0.405)	20.60*** (0.425)	5.553*** (0.0516)	5.234*** (0.0464)	0.318*** (0.0536)
Dependent variable	AR/S	AP/S	Net AR/S	AR/S	AP/S	Net AR/S
Country	U.S.	U.S.	U.S.	China	China	China
N	39860	39860	39860	667262	667262	667262
AR2	0.360	0.195	0.271	0.112	0.0712	0.0264

Notes: The dependent variables for the regressions are the ratio of accounts receivable to sales in columns (1) and (4), the ratio of accounts payable to sales in columns (2) and (5), and the ratio of net accounts receivable to sales in columns (3) and (6). Columns (1)-(3) use data for U.S. firms, and columns (4)-(6) use data for Chinese firms. All regressions include a set of sector-times-year fixed effects and a set of dummies of firm types. The U.S. data contain all non-financial firms. Chinese data contain only private firms.

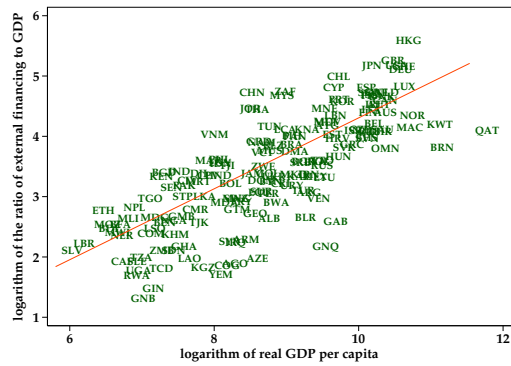


Figure A1: Cross-country income differences and financial development

Notes: This figure shows the cross-country correlation between the logarithm of GDP per capita (x axis) and the development of financial market (y axis). The level of financial market development is measured by the ratio of external financing to GDP, which is computed using the Financial Development and Structure Dataset (see Čihák et al., 2012) following the definition in Buera, Kaboski and Shin (2011). GDP per capita is computed using data from the Penn World Table 8.0.

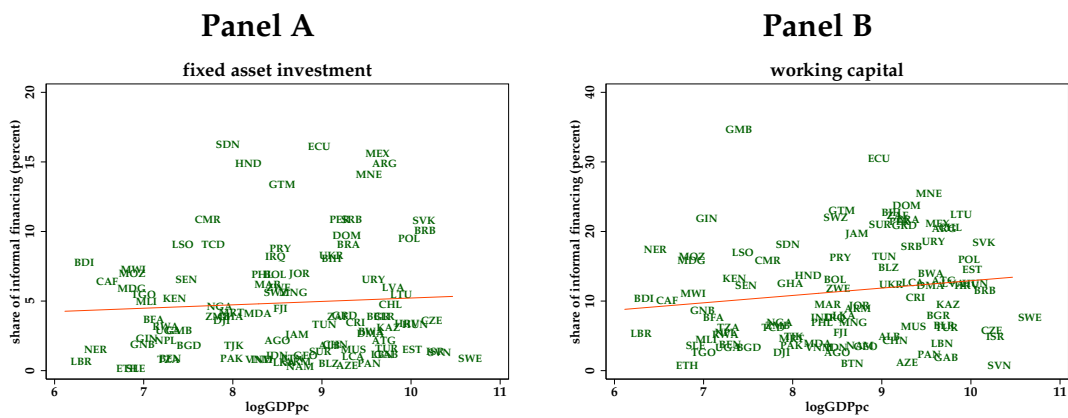


Figure A2: Share of informal financing; supplier credit only

Notes: This figure shows the correlation between the logarithm of real GDP per capita (x axis) and the share of informal financing (y axis) in fixed assets investment (Panel A) and in working capital (Panel B). Data for informal financing (supplier credit only) are taken from country-level financial indicators provided by the World Bank, and data for real GDP per capita are calculated using the Penn World Table.

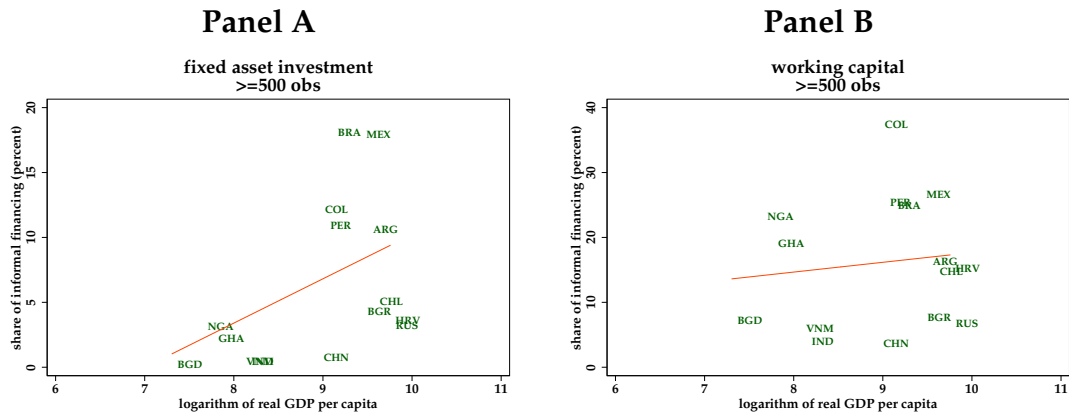
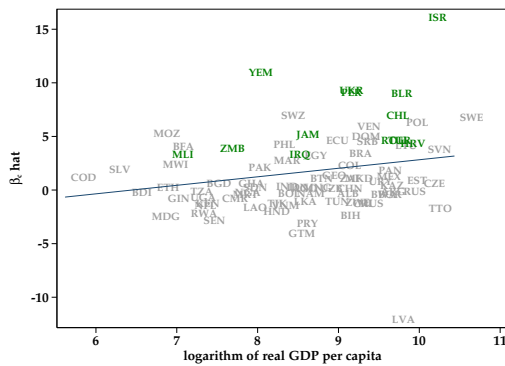


Figure A3: **Share of informal financing; countries with ≥ 500 observations**

Notes: This figure shows the correlation between the logarithm of real GDP per capita (x axis) and the share of informal financing (y axis) in fixed assets investment (Panel A) and in working capital (Panel B). Data for informal financing are calculated using the World Bank Enterprise Survey, and real GDP per capita is calculated using the Penn World Table. We exclude the countries with less than 500 firm-level observations.

Panel A: Fixed asset investment



Panel B: Working capital

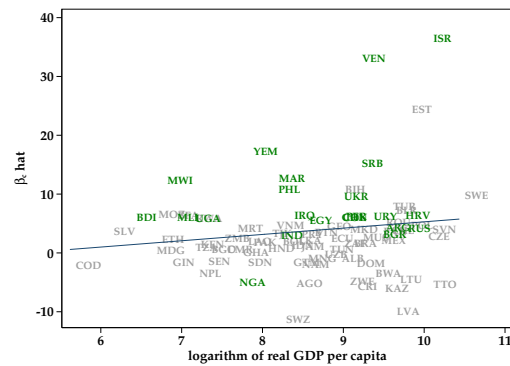


Figure A4: Substitutability of informal to formal financing increases with income

Notes: This figure shows the correlation between the logarithm of real GDP per capita (x axis) and the estimated coefficient $\hat{\beta}_c$ (y axis) (see regression equation 1). Each point in the figure represents one country. The figures plot all the regressions: the green ones are significant at the 5 percent level.

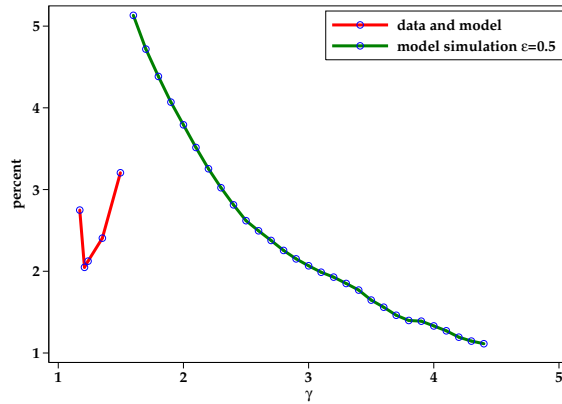


Figure A5: **Non-monotonicity of output gain from informal financing**

Notes: This figure plots the dynamics of output gain from informal financing with financial development. The five points on the red line correspond to the five groups of countries in our sample (Table 4). The green line is the model simulation by taking the benchmark calibration (corresponds to the last point on the red line), set $\epsilon = 0.5$, and gradually increase γ .

C Proofs

C.1 Without informal financing

Consider an entrepreneur household (a, z_1, z_2) . Without the chance of engaging in informal financing, the two members of the household make production decisions separately and their optimization problems read

$$\begin{aligned} \max_{k_1, l_1} \quad & Az_1 k_1^\alpha l_1^\chi - (r + \delta)k_1 - wl_1 \text{ s.t. } k_1 \leq \gamma a_1, \\ \max_{k_2, l_2} \quad & Az_2 k_2^\alpha l_2^\chi - (r + \delta)k_2 - wl_2 \text{ s.t. } k_2 \leq \gamma a_2. \end{aligned}$$

The unconstrained solution to the above problem is

$$\begin{aligned} k_1 &= [Az_1 (\frac{\alpha}{r + \delta})^{1-\chi} (\frac{\chi}{w})^\chi]^{-\frac{1}{1-\alpha-\chi}}, \\ l_1 &= [Az_1 (\frac{\alpha}{r + \delta})^\alpha (\frac{\chi}{w})^{1-\alpha}]^{-\frac{1}{1-\alpha-\chi}}, \\ k_2 &= [Az_2 (\frac{\alpha}{r + \delta})^{1-\chi} (\frac{\chi}{w})^\chi]^{-\frac{1}{1-\alpha-\chi}}, \\ l_2 &= [Az_2 (\frac{\alpha}{r + \delta})^\alpha (\frac{\chi}{w})^{1-\alpha}]^{-\frac{1}{1-\alpha-\chi}}. \end{aligned}$$

The unconstrained profits are

$$\begin{aligned} \pi_1(a_1, z_1) &= (Az_1)^{-\frac{1}{1-\alpha-\chi}} (\frac{\alpha}{r + \delta})^{-\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi}{w})^{-\frac{\chi}{1-\alpha-\chi}} (1 - \alpha - \chi), \\ \pi_2(a_2, z_2) &= (Az_2)^{-\frac{1}{1-\alpha-\chi}} (\frac{\alpha}{r + \delta})^{-\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi}{w})^{-\frac{\chi}{1-\alpha-\chi}} (1 - \alpha - \chi), \\ \pi(a, z_1, z_2) &= [(Az_1)^{-\frac{1}{1-\alpha-\chi}} + (Az_2)^{-\frac{1}{1-\alpha-\chi}}] (\frac{\alpha}{r + \delta})^{-\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi}{w})^{-\frac{\chi}{1-\alpha-\chi}} (1 - \alpha - \chi). \end{aligned}$$

Next consider the case where $a_1 = a_2 = \frac{1}{2}a$ and $z_2 \geq z_1$. The solution to the entrepreneurs' problem can be analyzed in the following three cases.

Case 1 If $\frac{1}{2}\gamma a \geq [Az_2 (\frac{\alpha}{r+\delta})^{1-\chi} (\frac{\chi}{w})^\chi]^{-\frac{1}{1-\alpha-\chi}}$, it holds that $\frac{1}{2}\gamma a \geq [Az_1 (\frac{\alpha}{r+\delta})^{1-\chi} (\frac{\chi}{w})^\chi]^{-\frac{1}{1-\alpha-\chi}}$ because $z_2 \geq z_1$. In this case, both entrepreneurs are unconstrained; therefore

$$\begin{aligned} \pi(a, z_1, z_2) &= [(Az_1)^{-\frac{1}{1-\alpha-\chi}} + (Az_2)^{-\frac{1}{1-\alpha-\chi}}] (\frac{\alpha}{r + \delta})^{-\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi}{w})^{-\frac{\chi}{1-\alpha-\chi}} (1 - \alpha - \chi), \\ \pi_a(a, z_1, z_2) &= 0. \end{aligned}$$

Case 2 If $\frac{1}{2}\gamma a \geq [Az_1 (\frac{\alpha}{r+\delta})^{1-\chi} (\frac{\chi}{w})^\chi]^{-\frac{1}{1-\alpha-\chi}}$ and $\frac{1}{2}\gamma a < [Az_2 (\frac{\alpha}{r+\delta})^{1-\chi} (\frac{\chi}{w})^\chi]^{-\frac{1}{1-\alpha-\chi}}$, in this case entrepreneur z_1 achieved unconstrained production scale, whereas entrepreneur

z_2 is constrained, such that

$$\begin{aligned}
\pi(a, z_1, z_2) &= (Az_1)^{\frac{1}{1-\alpha-\chi}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha-\chi}} \left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\alpha-\chi}} (1-\alpha-\chi) \\
&\quad + Az_2 k_2^\alpha \left(\frac{\chi Az_2 k_2^\alpha}{w}\right)^{\frac{\chi}{1-\chi}} - (r+\delta)k_2 - w \left(\frac{\chi Az_2 k_2^\alpha}{w}\right)^{\frac{1}{1-\chi}} \\
&= (Az_1)^{\frac{1}{1-\alpha-\chi}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha-\chi}} \left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\alpha-\chi}} (1-\alpha-\chi) \\
&\quad + (Az_2)^{\frac{1}{1-\chi}} \left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} k_2^{\frac{\alpha}{1-\chi}} - (Az_2)^{\frac{1}{1-\chi}} \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}} k_2^{\frac{\alpha}{1-\chi}} \\
&\quad - (r+\delta)k_2 \\
&= (Az_1)^{\frac{1}{1-\alpha-\chi}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha-\chi}} \left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\alpha-\chi}} (1-\alpha-\chi) \\
&\quad + (Az_2)^{\frac{1}{1-\chi}} \left[\left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}}\right] \left(\frac{1}{2}\gamma a\right)^{\frac{\alpha}{1-\chi}} \\
&\quad - (r+\delta)\frac{1}{2}\gamma a. \\
\pi_a(a, z_1, z_2) &= \frac{\alpha}{1-\chi} (Az_2)^{\frac{1}{1-\chi}} \left[\left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}}\right] \left(\frac{1}{2}\gamma\right)^{\frac{\alpha}{1-\chi}} a^{\frac{\alpha+\chi-1}{1-\chi}} \\
&\quad - (r+\delta)\frac{1}{2}\gamma.
\end{aligned}$$

Case 3 If $\frac{1}{2}\gamma a < [Az_1(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^\chi]^{\frac{1}{1-\alpha-\chi}}$, both entrepreneurs are constrained; therefore, the profit functions and the gradient of the profit function read

$$\begin{aligned}
\pi(a, z_1, z_2) &= [(Az_1)^{\frac{1}{1-\chi}} + (Az_2)^{\frac{1}{1-\chi}}] \left[\left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}}\right] \left(\frac{1}{2}\gamma a\right)^{\frac{\alpha}{1-\chi}} - (r+\delta)\gamma a, \\
\pi_a(a, z_1, z_2) &= \frac{\alpha}{1-\chi} [(Az_1)^{\frac{1}{1-\chi}} + (Az_2)^{\frac{1}{1-\chi}}] \left[\left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}}\right] \left(\frac{1}{2}\gamma\right)^{\frac{\alpha}{1-\chi}} a^{\frac{\alpha+\chi-1}{1-\chi}} - (r+\delta)\gamma.
\end{aligned}$$

C.2 With informal financing

The optimization problem of the individuals can be written as

$$\begin{aligned}
\pi(a, z_1, z_2) &= \max Az_1 k_1^\alpha l_1^\chi + Az_2 k_2^\alpha l_2^\chi - (r+\delta)(k_1 + k_2) - w(l_1 + l_2) \\
s.t. \quad &k_1 + k_2 \leq \gamma(a_1 + a_2).
\end{aligned}$$

The unconstrained solutions to the above problem are

$$\begin{aligned}
k_1 &= [Az_1(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^\chi]^{\frac{1}{1-\alpha-\chi}}, \\
l_1 &= [Az_1(\frac{\alpha}{r+\delta})^\alpha(\frac{\chi}{w})^{1-\alpha}]^{\frac{1}{1-\alpha-\chi}}, \\
k_2 &= [Az_2(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^\chi]^{\frac{1}{1-\alpha-\chi}}, \\
l_2 &= [Az_2(\frac{\alpha}{r+\delta})^\alpha(\frac{\chi}{w})^{1-\alpha}]^{\frac{1}{1-\alpha-\chi}}.
\end{aligned}$$

It follows that the profit function of the unconstrained solution can be written as

$$\begin{aligned}
\pi(a, z_1, z_2) &= Az_1 k_1^\alpha l_1^\chi + Az_2 k_2^\alpha l_2^\chi - (r+\delta)(k_1+k_2) - w(l_1+l_2) \\
&= [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_2)^{\frac{1}{1-\alpha-\chi}}] (\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} \\
&\quad - (r+\delta) [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_2)^{\frac{1}{1-\alpha-\chi}}] (\frac{\alpha}{r+\delta})^{\frac{1-\chi}{1-\alpha-\chi}} (\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} \\
&\quad - w [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_2)^{\frac{1}{1-\alpha-\chi}}] (\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi}{w})^{\frac{1-\alpha}{1-\alpha-\chi}} \\
&= [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_2)^{\frac{1}{1-\alpha-\chi}}] \\
&\quad [(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} - (\frac{\alpha^{1-\chi}}{(r+\delta)^\alpha})^{\frac{1}{1-\alpha-\chi}} (\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} \\
&\quad - (\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi^{1-\alpha}}{w^\chi})^{\frac{1}{1-\alpha-\chi}}] \\
&= [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_2)^{\frac{1}{1-\alpha-\chi}}] (\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}} (\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} (1-\alpha-\chi).
\end{aligned}$$

The FOCs of the constrained solution can be written as

$$\begin{aligned}
k_1 : A\alpha z_1 k_1^{\alpha-1} l_1^\chi &= r+\delta+\mu, \\
k_2 : A\alpha z_2 k_2^{\alpha-1} l_2^\chi &= r+\delta+\mu, \\
l_1 : A\chi z_1 k_1^\alpha l_1^{\chi-1} &= w, \\
l_2 : A\chi z_2 k_2^\alpha l_2^{\chi-1} &= w.
\end{aligned}$$

Rewrite FOCs w.r.t. l_1 and l_2 as $l_1 = (\frac{\chi A z_1 k_1^\alpha}{w})^{\frac{1}{1-\chi}}$ and $l_2 = (\frac{\chi A z_2 k_2^\alpha}{w})^{\frac{1}{1-\chi}}$. Take them back to the FOCs w.r.t. to k_1 and k_2 , and we have

$$\begin{aligned}
k_1^{\frac{1-\alpha-\chi}{1-\chi}} &= (Az_1)^{\frac{1}{1-\chi}} (\frac{\alpha}{r+\delta+\mu}) (\frac{\chi}{w})^{\frac{\chi}{1-\chi}}, \\
k_2^{\frac{1-\alpha-\chi}{1-\chi}} &= (Az_2)^{\frac{1}{1-\chi}} (\frac{\alpha}{r+\delta+\mu}) (\frac{\chi}{w})^{\frac{\chi}{1-\chi}}.
\end{aligned}$$

The above two equations give the capital ratio as $\frac{k_1}{k_2} = (\frac{z_1}{z_2})^{\frac{1}{1-\alpha-\chi}}$. Since in this

case the constraint $k_1 + k_2 \leq \gamma(a_1 + a_2)$, we can compute

$$\begin{aligned} k_1 &= \frac{\hat{z}}{1 + \hat{z}} \gamma(a_1 + a_2), \\ k_2 &= \frac{1}{1 + \hat{z}} \gamma(a_1 + a_2), \end{aligned}$$

where $\hat{z} = (\frac{z_1}{z_2})^{\frac{1}{1-\alpha-\chi}}$, it still holds that $l_1 = (\frac{\chi A z_1 k_1^\alpha}{w})^{\frac{1}{1-\chi}}$ and $l_2 = (\frac{\chi A z_2 k_2^\alpha}{w})^{\frac{1}{1-\chi}}$. We can then compute the profit function with constraint as

$$\begin{aligned} \pi(a, z_1, z_2) &= A z_1 k_1^\alpha l_1^\chi + A z_2 k_2^\alpha l_2^\chi - (r + \delta)(k_1 + k_2) - w(l_1 + l_2) \\ &= A z_1 k_1^\alpha \left(\frac{\chi A z_1 k_1^\alpha}{w}\right)^{\frac{\chi}{1-\chi}} + A z_2 k_2^\alpha \left(\frac{\chi A z_2 k_2^\alpha}{w}\right)^{\frac{\chi}{1-\chi}} \\ &\quad - (r + \delta)(k_1 + k_2) \\ &\quad - w \left(\frac{\chi A z_1 k_1^\alpha}{w}\right)^{\frac{1}{1-\chi}} - w \left(\frac{\chi A z_2 k_2^\alpha}{w}\right)^{\frac{1}{1-\chi}} \\ &= (A z_1)^{\frac{1}{1-\chi}} \left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} k_1^{\frac{\alpha}{1-\chi}} + (A z_2)^{\frac{1}{1-\chi}} \left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} k_2^{\frac{\alpha}{1-\chi}} \\ &\quad - (r + \delta)(k_1 + k_2) \\ &\quad - (A z_1)^{\frac{1}{1-\chi}} \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}} k_1^{\frac{\alpha}{1-\chi}} - (A z_2)^{\frac{1}{1-\chi}} \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}} k_2^{\frac{\alpha}{1-\chi}} \\ &= [(A z_1)^{\frac{1}{1-\chi}} \left(\left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}}\right)] k_1^{\frac{\alpha}{1-\chi}} \\ &\quad + [(A z_2)^{\frac{1}{1-\chi}} \left(\left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}}\right)] k_2^{\frac{\alpha}{1-\chi}} \\ &\quad - (r + \delta)(k_1 + k_2). \end{aligned}$$

Take the equations with k_1 and k_2 back to the above equations, and we get

$$\begin{aligned} \pi(a, z_1, z_2) &= [(A z_1)^{\frac{1}{1-\chi}} \left(\frac{\hat{z}}{1 + \hat{z}}\right)^{\frac{\alpha}{1-\chi}} + (A z_2)^{\frac{1}{1-\chi}} \left(\frac{1}{1 + \hat{z}}\right)^{\frac{\alpha}{1-\chi}}] \\ &\quad \left[\left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}}\right] \gamma^{\frac{\alpha}{1-\chi}} a^{\frac{\alpha}{1-\chi}} - (r + \delta) \gamma a \end{aligned}$$

Denote $\mathbb{B} = [(A z_1)^{\frac{1}{1-\chi}} \left(\frac{\hat{z}}{1 + \hat{z}}\right)^{\frac{\alpha}{1-\chi}} + (A z_2)^{\frac{1}{1-\chi}} \left(\frac{1}{1 + \hat{z}}\right)^{\frac{\alpha}{1-\chi}}] \left[\left(\frac{\chi}{w}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{\chi}{w^\chi}\right)^{\frac{1}{1-\chi}}\right] \gamma^{\frac{\alpha}{1-\chi}}$ and $\mathbb{C} = (r + \delta) \gamma$. Then we can write

$$\begin{aligned} \pi(a, z_1, z_2) &= \mathbb{B} a^{\frac{\alpha}{1-\chi}} - \mathbb{C} a, \\ \pi_a(a, z_1, z_2) &= \frac{\alpha}{1 - \chi} \mathbb{B} a^{\frac{\alpha + \chi - 1}{1 - \chi}} - \mathbb{C}. \end{aligned}$$