Financial Development Beyond the Formal Financial Market*

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Abstract

This paper studies the effects of financial development, taking into account both formal and informal financing. Motivated by empirical evidence, we build a general equilibrium model with heterogeneous entrepreneurs and the coexistence of the two types of financing. With the improvement of the formal financial market, the supply of informal financing increases, while the demand for it declines, generating a hump-shaped pattern in the incidence of informal financing over the development process. Our quantitative analysis shows that informal financing contributes more to the aggregate output of the richer countries in our sample than poorer countries, which suggests that at the early stage of economic development, the output loss from financial frictions could be reinforced by the low supply of informal financing.

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1 Introduction

Since Schumpeter (1911), economists have argued that well-developed financial markets are crucial for economic growth. Papers in the financial development literature use a variety of indicators to measure financial market development. For example, Greenwood, Sanchez and Wang (2010) use interest rate spreads to measure the effectiveness of a financial market. Djankov, McLiesh and Shleifer (2007) construct an indicator called "private credit," which includes loans issued by commercial banks and other financial institutions to the private sector. Buera, Kaboski and Shin (2011) use "external financing," which, in addition to "private credit," also includes funds obtained by the private sector from bond and equity markets.

However, these indicators suffer from one key caveat: they only measure formal financing activities in the economy, and therefore exclude financing from non-financial lenders, such as moneylenders, friends, family, and input suppliers. These types of loans tend to be relationship- and reputation-based, unregulated, and most likely do not appear on a firm's balance sheet. They are inherently difficult to measure, especially at the aggregate level. We label them as *informal* financing, in contrast with the *formal* financing provided by financial intermediaries and financial markets.

One might expect poor countries to rely on informal financing more than rich countries as a mitigation of the losses from financial frictions. If so, the importance of a well-developed formal financial market might be overstated in traditional analyses. However, we show that informal financing as a share of total financing and the average size of informal financing loans are both higher in rich countries than in poor countries. Similarly, we find that financially constrained firms in richer coun-

tries use relatively more informal financing than their constrained counterparts in poorer countries, suggesting a greater substitution of informal for formal financing in rich countries.

We show that this empirical pattern can be generated by a simple model of heterogeneous entrepreneurs facing financial frictions amid the coexistence of both formal and informal financing. The intuition is simple: consider an entrepreneur who needs to finance production but has limited access to formal financing. Potential informal lenders have a comparative advantage in lending to her because of better information about the quality of her projects or better enforcement over repayment of loans. However, unlike formal lenders, informal lenders are themselves faced with financial constraints. As such, the capacity of informal lenders to provide loans will be lower in poorer countries because of greater financial frictions and lower average wealth for use as collateral by informal lenders. Furthermore, to a certain degree, the lenders of informal and formal financing are faced with the same fundamental frictions in the economy. As a result, the frictions that lead to a less efficient formal financial market in poor countries would also lower the supply of informal financing. Even though the need for informal financing by entrepreneurs maybe greater in poor countries, the equilibrium quantity of informal loans may actually be lower than richer countries. Thus the presence of informal financing amplifies the impacts of frictions in formal financial markets.

In the model, there is a continuum of islands, each of which is populated by workers and heterogeneous entrepreneurs with different productivity and wealth. All entrepreneurs have access to an economy-wide formal financial market. The size of formal loans is limited by a collateral constraint, which can be relaxed with the development of the formal financial market and the accumulation of wealth. Entrepreneurs from the same island can also borrow from each other through an infor-

mal channel subject to a search friction. Informal financing facilitates the movement of resources to more productive entrepreneurs on the island who are constrained in the formal financial market. The demand for informal loans declines when the formal market becomes more efficient. The supply of informal loans, however, is determined by the ability of less productive entrepreneurs to access formal loans, which increases with their wealth and the efficiency of the financial market. Therefore, when the supply-side force dominates, the incidence of informal financing could increase with economic development.

We calibrate our model to match five countries: Mexico, Argentina, Colombia, Nigeria and Kenya. The country-specific calibration matches well the aggregate moments such as GDP per capita, ratio of external financing to GDP and informal financing as a share of total financing, and it delivers a distribution of firm size and informal financing that is consistent with the firm-level data. The quantitative results show that informal financing plays a more important role in the middle-income countries (Mexico, Argentina and Colombia) than in the low-income countries (Nigeria and Kenya). Informal financing accounts for 5.7 percent of Mexico's GDP, and in contrast, it contributes 4.3 percent and 3.2 percent of the GDP of Nigeria and Kenya, respectively.

Literature review This paper belongs to several strands of literature. First, it contributes to the empirical literature that studies informal financing and firm performance. This strand of literature often takes firm-level data from a specific country and studies the role of informal financing for firms with limited access to formal financing. The results are rather inconclusive. Take the studies on informal financing in China as an example: while Allen, Qian and Qian (2005) show that informal financing is important to promote growth in China, Ayyagari, Demirgüç-Kunt

and Maksimovic (2010) find that firms with access to formal credit (bank loans) grow faster than firms that utilize only informal financing. Degryse, Lu and Ongena (2013) on the other hand show that informal financing that is simultaneously granted with formal financing contributes to firm growth. Our paper contributes to this literature by focusing on the cross-country differences in informal financing and emphasizing the relationship among informal financing, formal financing, and economic development.[1]

Second, our paper is one of a few that models explicitly the interaction between formal and informal financing (see Karaivanov and Kessler, 2018 and Madestam, 2014). Similar to our paper, Madestam (2014) provides a model of informal financing and generates the substitution between informal and formal financing in equilibrium.[2] This paper differs from Madestam (2014) in two dimensions. First, in Madestam (2014) the degree of substitutability between the two types of financing is determined by the monopolistic power of the formal lenders, while in this paper, it is determined by the informal lenders' access to formal financing. This difference allows us to link the substitutability to the level of economic development. Second, our paper builds informal financing into a quantitative framework to examine the aggregate impact of informal financing.

Lastly, our paper belongs the strand of literature devoted to quantifying the impact of financial frictions on aggregate productivity. It is widely understood that resource misallocation across firms can account for a large fraction of productivity differences across countries (Hsieh and Klenow, 2009, Guner, Ventura and Xu, 2008

[1] Allen, Qian and Xie (2018) also exploits cross-region differences in China and documents that certain type of informal financing is also more prevalent in regions with better access to formal financing.

[2] Both papers borrow insights from the literature on trade credit (see Biais and Gollier, 1997 and Burkart and Ellingsen, 2004) that the existence of informal financing reflects a certain comparative advantage of informal lenders in extending loans to borrowers.

and Restuccia and Rogerson, 2008). Many papers have since shown that financial frictions lead to resource misallocation and have quantified the aggregate productivity loss from financial frictions (see Buera, Kaboski and Shin, 2011, Greenwood, Sanchez and Wang, 2010, Midrigan and Xu, 2014 and Moll, 2014). Our paper extends this literature by incorporating informal financing into the framework and quantifying its importance.

2 Empirical evidence

In this section, we combine several databases to document the empirical patterns of cross-country differences in informal financing at the country level (section 2.1) and at the firm level (section 2.2). The types of informal financing included in the empirical exercise share two characteristics: first, the lenders are not specialized financial intermediaries, and thus do not have access to a deep sources of funding, and second, they are excluded in the measures of (formal) financial development commonly used in the literature.

2.1 Aggregate-level pattern

As shown in Figure A1, there is a positive correlation between real GDP per capita (averaged over 2000–2011) and the level of formal financial market development, measured as the ratio of external financing to GDP (averaged over 2000–2011) in a sample of 136 countries. Conventional wisdom says that poor countries might use relatively more informal financing than rich countries because they are more constrained in the formal financial market. However, as shown in Figure 1, in this

sample of 109 countries in the World Bank Enterprise Survey (WBES), the share of informal financing used to finance fixed asset investment and working capital in fact increases slightly with the income level of the countries.[3]

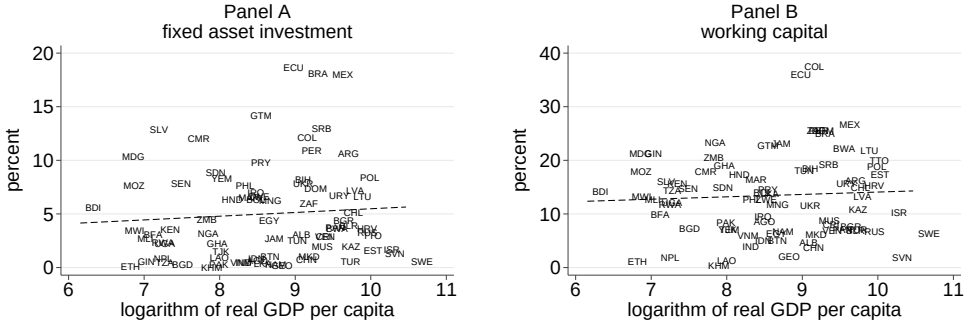

Figure 1: **Share of informal financing across countries**

Sources: WBES (2009-2014), PWT 9.1.
Notes: This figure shows the correlation between the logarithm of real GDP per capita (x axis) and the share of informal financing (y axis) in fixed assets investment (Panel A) and in working capital (Panel B).

Figure 2 shows cross-country differences in the average magnitude of informal financing based on a sample of 45 countries from the Global Entrepreneurship Monitor (GEM) 2015 APS individual database.[4] Panel A shows that the share of population who lent money to friends, family members and coworkers to finance their businesses is higher in poor countries than rich countries. However, the average

[3]The empirical evidence are based on WBES standardized dataset (2006–14). The share of fixed asset investment financed by informal loans is the sum of variables k5f (purchase on credit from suppliers and advances from customers) and k5hd (loans from moneylenders, friends, relatives, etc.). The share of working capital financed by informal loans is equal to the sum of variables k3f (purchases on credit from suppliers and advances from customers) and k3hd (loans from moneylenders, friends, relatives, etc.). We drop all establishment-level observations and all observations with missing values. We only include the surveys with more than 100 observations. Summary statistics of the variables by country can be found in Table A1. Country-level statistics are computed as the weighted average of firms.

[4]The evidence in based on GEM 2015 APS individual database. We identify the informal lenders (individuals who lent money to friends, family members and coworkers in the past three years) and the size of informal financing using variable *bafundus* (size of informal financing in US dollars) and *barel* (relationship to the borrower). The country-level statistics are computed as the weighted average of individuals in each country.

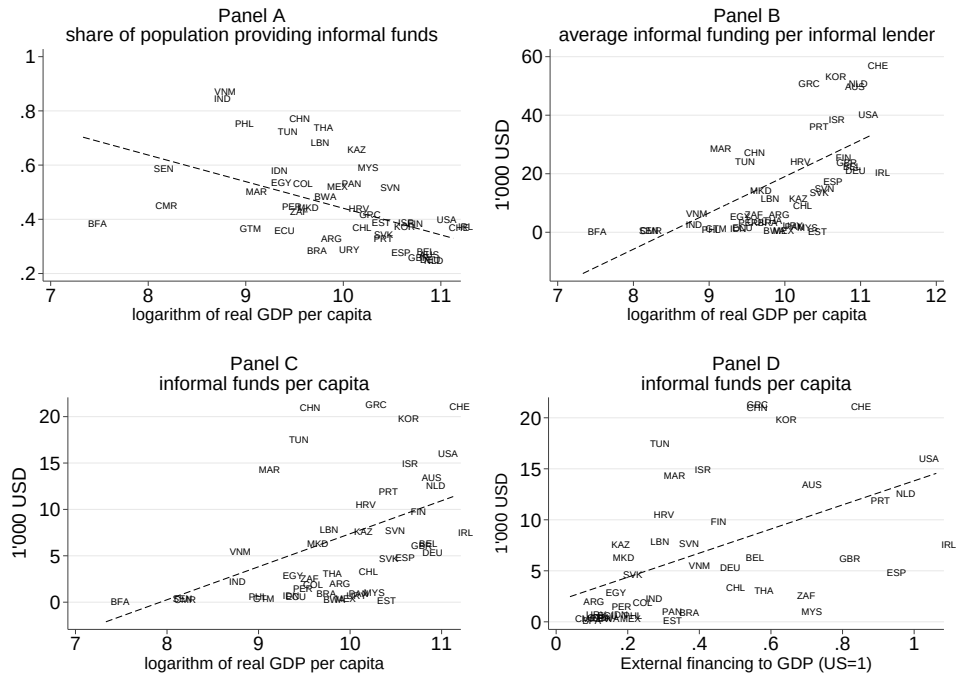

Figure 2: **Magnitude of information financing across countries**

Source: GEM APS individual dataset 2015, PWT 9.1, Financial Development and Structure Database.
Notes: The country-level indicators used here are weighted averages of individuals in the 2015 GEM APS database. Informal financing refers to money lent to friends, family members and coworkers. Panel B plots the average size of loans informal lenders provided to informal debtors. Panel C and D plot the amount of informal loans provided by an individual in each country, which is computed as the product of the share of population lending to their family, friends and coworkers (Panel A) and the average size of the informal loan (Panel B).

informal lender in rich countries provides greater total funding than poor countries (Panel B). Consequently, total informal financing per capita is greater in richer countries (Panel C) and in countries with a better developed formal financial market (Panel D).

2.2 Firm-level pattern

In this section, we study the substitutability of informal financing for formal financing in different countries at the firm level.

For each country c in WBES, we pool the surveys from different years, and run the following regression:

$$inf_{ist} = \alpha + \beta_c I\_constrained_i + \chi_{st} + I\_young_i \times I\_small_i + \gamma_i + \varepsilon_{ist}. \quad (1)$$

In the regression,

- inf_{ist} is the percent of either fixed assets investment or working capital of firm i in sector s of year t that is financed through informal channels.[5]
- $I\_constrained_i$ is a dummy indicator of whether the firm i is financially constrained. A firm is defined as being financially constrained if it reports that access to finance is its biggest obstacle of growth.
- χ_{st} is a set of sector $\times$ year fixed effects.
- I_young_i is a dummy indicator of whether the firm is young ($\leq$ 5 years old).
- I_small_i is a dummy indicator of whether the firm is small ($\leq$ 10 employees).
- γ_i is a dummy indicator of firm i's type: whether it is government-owned, private, or foreign.

The estimated coefficient $\hat{\beta}_c$ is the object of interest. In country c, compared with financially unconstrained firms, $\hat{\beta}_c$ percent more fixed asset investment or working capital of the constrained firms is financed through informal channels. We interpret

[5]The share of fixed assets investment financed through informal channels is computed as the sum of variables k5f (purchase on credit from suppliers and advances from customers) and k5hd (loans from moneylenders, friends, relatives, etc.) and the share of working capital financed through informal channels is computed as the sum of variables k3f (purchases on credit from suppliers and advances from customers) and k3hd (loans from moneylenders, friends, relatives, etc.). See footnote 3 for the details of the treatment of data.

$\hat{\beta}_c$ as the substitutability of informal financing for formal financing at the firm-level in country c.

In Figure 3, we plot the estimated coefficient $\hat{\beta}_c$ against the GDP per capita of country c. The dependent variables in Panel A and Panel B are the percent of informal financing in fixed assets investment and that in working capital, respectively. In both panels, $\hat{\beta}_c$ is positive for almost all countries and $\hat{\beta}_c$ increases with the income level of the countries. This means that financially constrained firms in developed countries rely more on informal channels to finance their production than do their financially constrained counterparts in developing countries.

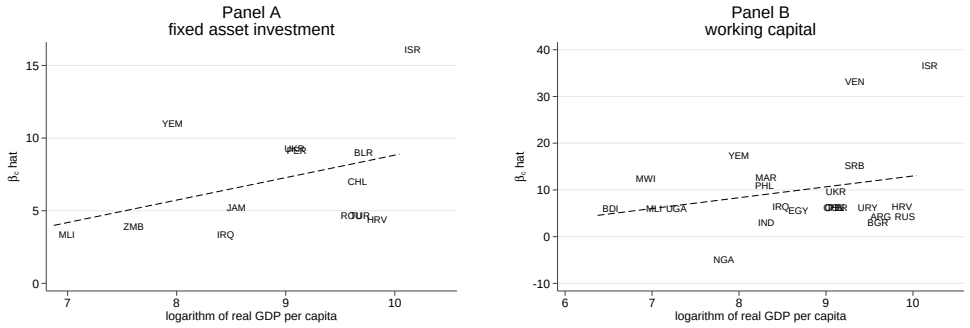

Figure 3: **Substitutability of informal for formal financing**

Notes: This figure shows the correlation between real GDP per capita (x axis) and the estimated coefficient $\hat{\beta}_c$ (y axis) (see regression equation 1) across countries. The figure plots the countries with an estimated $\hat{\beta}_c$ that is significant at the 5 percent level.

Similar results emerge when we run a cross-country regression by pooling firms from different countries in the WBES database:

$$inf_{ist} = \alpha + \beta_1 I\_constrained_i + \beta_2 I\_constrained_i \times log\ GDPpc_c$$
$$+\beta_3 log\ GDPpc_c + \chi_{st} + I\_young_i \times I\_small_i + \gamma_i + \varepsilon_{ist}, \quad (2)$$

where $log\ GDP\ pc_c$ is the logarithm of real GDP per capital of country c. Table 1 shows that the interaction term between being financial constrained and log GDP

per capita is positive and significant (column 1 and 2). In addition, column 3 and 4 show that the interaction term between financial constraint and log external financing to GDP is also positive and significant, which suggests that the substitutability of informal for formal financing at the firm level increases both with income and the development of formal financial market.[6]

Table 1: **Substitutability of informal for formal financing (pooling)**

	(1)	(2)	(3)	(4)
financially constrained	-5.126***	-7.982	-4.139***	-3.527
	(1.662)	(5.529)	(1.256)	(3.422)
financial constraint X log GDP pc	0.796***	1.293**		
	(0.223)	(0.614)		
log GDP pc	0.556	-0.239		
	(0.354)	(0.848)		
financial constraint X log ext. fin to GDP			1.764***	1.966**
			(0.414)	(0.937)
log ext. fin to GDP			-0.159	-1.272
			(0.279)	(1.065)
Dependent variable	fixed assets inv.	working capital	fixed assets inv.	working capital
sectorXyear FE	Y	Y	Y	Y
firm-type FE	Y	Y	Y	Y
firm-size&age FE	Y	Y	Y	Y
N	30204	30204	29532	29532
AR2	0.0274	0.0559	0.0265	0.0563

Notes: The dependent variables for the regressions are the percent of informal financing in fixed assets investment (columns 1 and 3) and the percent of informal financing in working capital (columns 2 and 4). All regressions include a set of sector times year fixed effects, firm type fixed effects, and firm size and age fixed effects. Standard errors are clustered at the country- and sector-level.

2.3 Discussion

Taking stock, this section documents the following three facts about informal financing. First, at the country level, as the income of a country increases, both formal and informal financing increase (Figure A1, 1 and 2). Second, at the firm level, financially constrained firms use more informal financing compared with unconstrained

[6]The regressions in column 3 and 4 are $inf_{ist} = \alpha + \beta_1 I\_constrained_i + \beta_2 I\_constrained_i \times log\frac{ext.fin._c}{GDP_c} + \beta_3 log\frac{ext.fin._c}{GDP_c} + \chi_{st} + I\_young_i \times I\_small_i + \gamma_i + \varepsilon_{ist}$.

firms ($\hat{\beta}_c$ in Figure 3 is overwhelmingly positive across countries). Lastly, the substitutability between informal and formal financing at the firm level increases with economic development (see Figure 3 and Table 1). These three facts motivate the model in the following section.

3 Model

This section introduces a dynamic general equilibrium model with heterogeneous entrepreneurs who have access to frictional formal and informal financing.

3.1 Economic environment

Time is discrete and infinite. There is one good in the economy, which is used for consumption and investment.

There is a continuum of islands, each of which is populated by one household with two entrepreneurs and another household with N workers. The entrepreneurs use labor and capital to produce goods. The workers provide labor inelastically to the market and earn wages for their work. Unlike the entrepreneur households, the worker households do not have access to the capital market, i.e. they are "hand-to-mouth."

3.2 Preference, endowment, and production technology

The entrepreneurs operate a decreasing return to scale production technology that transforms capital and labor into the consumption/investment good, such that

$$y_t = Az_t k_t^{\alpha} l_t^{\chi},$$

where A is the economy-wide total factor productivity (TFP) and z_t is the idiosyncratic productivity shock faced by the entrepreneur, which follows an exogenous stochastic process.

For a worker household, the preference of its n^{th} member is time-separable with an instantaneous utility function of the CRRA form $u(c_{n,t}) = \frac{c_{n,t}^{1-\sigma}-1}{1-\sigma}$. The utility of the worker household over a sequence of consumption $c_n = \{c_{n,t}\}_{t=0}^{\infty}$ is

$$U^w(c_1, ..., c_N) = \sum_{t=0}^{\infty} \beta^t \sum_{n=1}^{N} u(c_{n,t}),$$

which means that the household puts the same weight on the welfare of its members.[7]

Similarly, for an entrepreneur household, the preference of the m^{th} member is time-separable with an instantaneous utility function of the CRRA form $u(c_{m,t}) = \frac{c_{m,t}^{1-\sigma}-1}{1-\sigma}$. The utility of the entrepreneur household over a sequence of consumption $c_m = \{c_{m,t}\}_{t=0}^{\infty}$ is

$$U^e(c_1, c_2) = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \sum_{m=1,2} u(c_{m,t}).$$

[7]The workers' wage is deterministic, therefore there is no expectation operator over the future utilities.

The expectation is taken over a stochastic stream of consumption $\{c_{m,t}\}_{t=0}^{\infty}$ and idiosyncratic productivity $\{z_{m,t}\}_{t=0}^{\infty}$.

3.3 Timing

At the beginning of period t, the entrepreneur households enter each period with wealth a_t, distribute the wealth to the two entrepreneurs in the household $(a_{1,t} + a_{2,t} = a_t)$ and send them out to produce. Without loss of generality, we assume that the entrepreneur households do not have information about the history and the realization of entrepreneurs' idiosyncratic productivity shocks. At the same time, the worker households send their members out to work. After the entrepreneurs' idiosyncratic productivity $z_{1,t}$ and $z_{2,t}$ are realized, they seek financing by going to the formal financial market to take out formal loans and, if the formal loans are insufficient, they search for the other entrepreneur from the same household to borrow from her informally. With probability $\epsilon \in [0, 1]$ the search is successful. Then production begins. At the end of production, the entrepreneurs repay their formal and informal loans. Then entrepreneurs and workers return to their households with their production profit and wage income, respectively. The households choose consumption and saving. An illustration of the timing can be found in Figure 4.

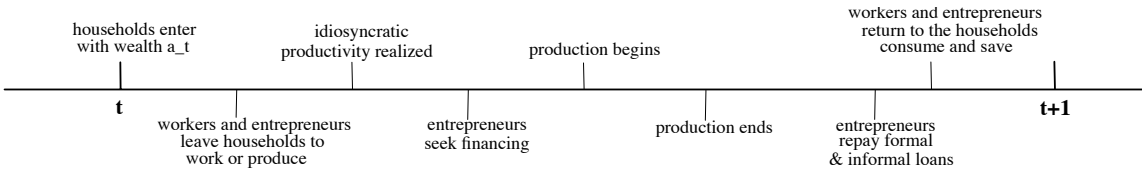

Figure 4: **Timing**

3.4 Markets and frictions

The workers in the economy are perfectly mobile across islands. There exists an economy-wide competitive labor market with wage w that clears the market.

There is an economy-wide competitive formal financial market. Following the literature, we model the formal financial market as a capital rental market, from which the entrepreneur households from all islands can save and borrow at a risk-free interest rate r.

The financial frictions in the economy originate from the limited enforcement over the repayment of formal loans. As a result, the entrepreneurs' borrowing from the formal financial market is limited by the amount of collateral they own. The "no default" formal loan contract requires that $\zeta k \leq a,$ where $\zeta < 1$ is the share of capital that entrepreneurs can run away with if they default on the contract. The size of formal loan is therefore constrained, such that $k \leq \gamma a$, where $\gamma = \frac{1}{\zeta}$.

Besides accessing the economy-wide formal financial market, entrepreneurs from the same island can also borrow from and lend to each other informally. We assume that the repayment of informal loans between members of the same island can be enforced perfectly. This comparative advantage allows informal financing to coexist with formal financing. But lenders of informal financing are not specialized financial intermediaries, and therefore they are subject to the same constraint on the formal financial market as borrowers are. More formally, and without loss of generality, let i be the potential lender of informal loan, $-i$ the potential borrower, and $\hat{k} \geq 0$ the size of the informal loan. Formal financial constraints limit the size of capital inputs and informal loan. That is, the following constraints need to be satisfied: $k_i + \hat{k} \leq \gamma a_i$ and $k_{-i} \leq \gamma a_{-i}$.

In addition, one might wonder whether the assumption of a perfect enforcement

of the repayment of informal loans is too strong. It is reasonable to assume that both formal and informal lending are affected by the quality of the institutions in the economy. For example, both formal and informal lenders could benefit from the protection of a more effective judicial system. To capture this idea, we assume entrepreneurs' search for informal financing is successful only with probability ϵ.[8]

The structure of the financial markets in this economy is illustrated in Figure 5.[9]

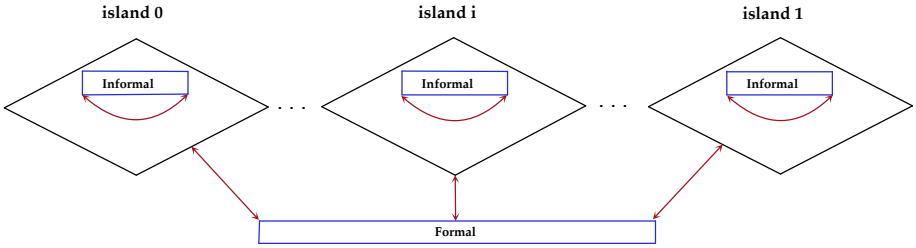

Figure 5: **Financial markets in the economy**

3.5 Discussion

Several assumptions of the model merit discussion. First, we use a decreasing returns to scale production function instead of constant returns to scale to better match firm heterogeneity in the data. Second, in order to keep the model tractable, we assume that the consumption and saving decisions are made at the household level to rule out multiple equilibria.[10] Third, we abstract from individual occupational

[8] Alternatively, we could also assume the enforcement of informal financing is imperfect but better than that of the formal financing.

[9] The model is akin to the island economy in Gertler and Kiyotaki (2010). The island-specific informal financial markets are analogous to the banks of Gertler and Kiyotaki (2010), and the economy-wide formal financial market is analogous to the inter-bank lending market.

[10] If entrepreneurs can make saving decisions on their own, there can be multiple equilibria in the dynamic game between the two entrepreneurs on the same island because the savings of the two entrepreneurs are substitutable to a certain degree.

choice (i.e. entrepreneur versus worker) because with occupational choices of multiple household members and a decreasing return to scale production technology, the household profit function can be convex-concave under some parameter values, which could lead to multiple equilibria in a dynamic model (see Skiba, 1978).

4 Recursive competitive equilibrium

This section presents the optimization problem faced by individuals in the economy and defines the recursive competitive equilibrium.

Workers supply one unit of labor inelastically and bring back to the household their wage w. Since the worker household is hand-to-mouth, they consume their wage every period, i.e. $c^w = w$.

Now consider the two entrepreneurs from the entrepreneur household of island i. Without loss of generality, we label them as i and $-i$ and assume that entrepreneur $-i$ is more productive than entrepreneur i, that is, $z_i < z_{-i}$. Therefore, entrepreneur i is the potential lender of informal financing on the island and entrepreneur $-i$ the potential borrower. Let $\pi(a, z_i, z_{-i}, \omega)$ be the aggregate profit function of the entrepreneur household in island i with wealth a and productivities z_i and z_{-i}. The state variable $\omega \in \{0, 1\}$ is an i.i.d. shock indicating whether the search for informal financing is successful.

If the search for informal financing fails ($\omega = 0$), the two entrepreneurs maximize their profit subject to their independent collateral constraints. The optimization problem of an entrepreneur with productivity $\tilde{z}$ and wealth $\tilde{a}$ reads

$$\tilde{\pi}(\tilde{z}, \tilde{a}) = \max_{k,l} A\tilde{z}k^{\alpha}l^{\chi} - (r + \delta)k - wl, \ s.t. \ k \leq \gamma\tilde{a}. \tag{3}$$

In this case, the total profit of the entrepreneur household is the sum of the profit of its two members: $\pi(a, z_i, z_{-i}, 0) = \tilde{\pi}(z_i, \frac{a}{2}) + \tilde{\pi}(z_{-i}, \frac{a}{2})$.[11] The aggregate profit function $\pi(a, z_i, z_{-i}, 0)$ can be solved analytically and is concave in household wealth a (see the details in Appendix B.1).

On the other hand, consider the case where the search for informal financing is successful ($\omega = 1$). Assume that the informal lender can make a take-it-or-leave-it offer to the informal borrower.[12] The optimization problem is equivalent to the lender maximizing the total profit of the two entrepreneurs subject to the financial constraints, such that

$$\begin{aligned} \pi(a, z_i, z_{-i}, 1) \quad = \quad & \max \ A z_i k_i^{\alpha} l_i^{\chi} + A z_{-i} (k_{-i} + \hat{k})^{\alpha} l_{-i}^{\chi} \\ & -(r+\delta)(k_i + k_{-i} + \hat{k}) - w(l_i + l_{-i}), \\ s.t. \qquad & k_i + \hat{k} \leq \gamma \frac{a}{2}, \ k_{-i} \leq \gamma \frac{a}{2}, \\ & k_i \geq 0, \ k_{-i} + \hat{k} \geq 0. \end{aligned} \tag{4}$$

where $\hat{k}$ is the size of informal financing. The profit function $\pi(a, z_i, z_{-i}, 1)$ can also be characterized analytically and is concave in household wealth a (see details in Appendix B.2).

Definition 1 *The recursive competitive equilibrium consists of prices (r, w), value function of the entrepreneur household $V^e(a, z_i, z_{-i}, \omega)$, policy functions of the entrepreneur household: consumption $c^e(a, z_i, z_{-i}, \omega)$, inputs $k_i(a, z_i, z_{-i}, \omega)$, $k_{-i}(a, z_i, z_{-i}, \omega)$, $\hat{k}(a, z_i, z_{-i}, \omega)$, $l_i(a, z_i, z_{-i}, \omega)$, $l_{-i}(a, z_i, z_{-i}, \omega)$, and next period wealth $a'(a, z_i, z_{-i}, \omega)$, the consumption of*

[11] Notice that since the division of wealth within the household happens before the realization of idiosyncratic productivity and the realization of the idiosyncratic shock is observable only to the entrepreneurs, household wealth a will be divided equally between the two entrepreneurs.

[12] The bargaining power between the lender and the borrower of informal financing does not affect the final result because the consumption and saving decisions are made at the household level.

workers c^w, and the stationary distribution of the entrepreneur households $\Omega(a, z_i, z_{-i}, \omega)$, such that

1. Given the prices, the policy functions of the entrepreneur household solve the production optimization problems 3 and 4, and

2. Given the prices, the value function and policy functions of the entrepreneur household solve the following problem,

$$\begin{aligned} V^e(a, z_i, z_{-i}, \omega) &= \max_{c_i, c_{-i}, a'} u(c_i) + u(c_{-i}) + \beta \mathbb{E}_{z_i', z_{-i}'} V^e(a', z_i', z_{-i}', \omega'), \\ s.t. \quad & c_i + c_{-i} + a' = \pi(a, z_i, z_{-i}, \omega) + (1 + r)a, a' \geq 0, \end{aligned}$$

   where the household profit function $\pi(a, z_i, z_{-i}, \omega)$ is characterized in Appendix B.1 and B.2.

3. Workers' consumption satisfies the budget constraint $c^w = w$.

4. The interest rate r clears the formal financial market and the wage w clears the labor market.

5. The distribution Ω is stationary, such that

$$\Omega(a', z_i', z_{-i}', \omega') = \int_{a, z_i, z_{-i}, \omega} \mathbb{I}_{a' = a'(a, z_i, z_{-i}, \omega)} \Upsilon(z_i', z_{-i}', \omega' | z_i, z_{-i}, \omega) d\Omega(a, z_i, z_{-i}, \omega),$$

   where $\mathbb{I}_{a' = a'(a, z_i, z_{-i}, \omega)}$ is an indicator function and $\Upsilon(z_i', z_{-i}', \omega' | z_i, z_{-i}, \omega)$ is the transition matrix of the exogenous state variables.

5 Quantitative analysis

In this section, we calibrate the model to match the Mexican data (section 5.1) and use the calibrated model for three quantitative analyses. In section 5.2, we study the aggregate dynamics of the development of the formal financial market, that is, a relaxation in the collateral constraint γ. In section 5.3, we compare the gain in aggregate output from informal financing for five countries at different stages of economic development. Section 5.4 shows the changes in aggregate output, informal financing and formal financing with a development in the formal financial market across countries.

5.1 Calibration to Mexico

Our benchmark calibration matches data moments of Mexico, the richest country in the WBES database with more than 500 observations. We normalize the TFP of Mexico to 1. We pick the inter-temporal elasticity of substitution σ to be 2 and calibrate β to match an annual risk-free interest rate of 4 percent. The measure of workers is set to $N = 18$, since the share of entrepreneurs in the data is around 10 percent and the measure of entrepreneurs in the model is 2. The collateral constraint parameter γ is calibrated to match the ratio of external financing to GDP. The probability of finding informal financing ϵ is calibrated to match the share of informal financing in the WBES data. We model the exogenous process of idiosyncratic productivity as a Poisson death shock with probability π and a redraw of the idiosyncratic productivity from a Pareto distribution with tail parameter μ. Following Buera, Kaboski and Shin (2011), we set the death shock probability $\pi = 0.1$ and we set the scale of pro-

Table 2: **Summary of calibration**

Parameter		Value	Target/Source	Data	Model
A	TFP	1	normalized to 1	–	–
α	capital share in the production function	0.28	capital share of $1/3$	–	–
π	Poisson death rate	0.1	Buera, Kaboski and Shin (2011)	–	–
$\alpha+\chi$	scale parameter in production function	0.85	Buera, Kaboski and Shin (2011)	–	–
N	measure of workers	18	share of entrepreneur	10%	10%
δ	capital depreciation rate	0.06	annual depreciation rate	6%	6%
β	discount rate	0.7	annual risk-free interest rate	4%	4%
μ	Pareto tail	3.6	top 25 percent employment share in WBES database	89%	88%
γ	collateral value	1.8	ratio of external financing to GDP	0.41	0.41
ϵ	probability of informal financing	0.38	investment and working cap. financed by informal financing	20.3%	20.3%

Notes: This table is the summary of calibration of the benchmark model to match the richest quintile of the countries. The share of investment and working capital financed by informal funds are computed as the average of these two.

duction function $\alpha+\chi=0.85$, and we calibrate μ to match the employment share of the top 25 percent Mexican firms in the WBES database. Table 2 shows a summary of the calibration.

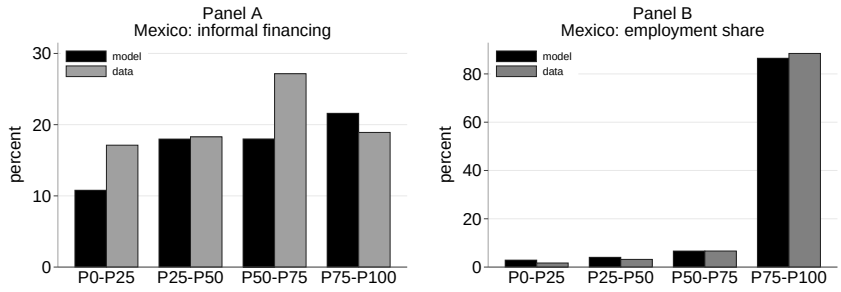

Figure 6: **Informal financing and employment share by firm size (non-targeted moments)**

Source: WBES (2009-2014), model output.
Notes: These figures plot the distribution of informal financing and employment share by firm size (from the smallest quartile to the largest quartile). In panel A, informal financing is computed as the average of the share of informal financing in fixed asset investment and working capital in WBES data, and it is computed as informal financing as a share of output in the model.

Figure 6 shows that the calibration also matches the distribution of informal financing and employment across firm quartiles. As shown in the left panel, in both the model and the data, larger firms obtain a greater portion of their financing from informal sources than small firms, which is a result of the assortative matching between informal lenders and borrowers. In the model, although poor entrepreneurs

are more constrained and demand more informal financing, they are matched with an informal lender who is also poor and therefore has less capacity to lend. Thus, poor entrepreneurs use less informal financing than richer entrepreneurs in equilibrium. The right panel of Figure 6 shows a skewed firm-size distribution in both the data and the model with the top quartile of Mexican firms constituting approximately 89% of aggregate employment.

5.2 The aggregate dynamics of financial development

In this section, we examine the aggregate dynamics of formal financial development by increasing the collateral constraint parameter γ in the calibrated model. The left panel of Figure 7 shows that the aggregate output is increasing and concave in γ. The increasing relationship comes entirely from a better allocation of resources across heterogeneous entrepreneurs as γ rises. The concavity of the relationship is a result of the decreasing returns to scale production technology, under which all entrepreneurs become unconstrained when the financial market is sufficiently developed. That is, the economy converges to a frictionless neoclassical economy when γ approaches infinity.

As shown in the right panel of Figure 7, the aggregate volume of formal financing follows a similar pattern as that of the aggregate output. However, the aggregate volume of informal financing first increases with γ, peaks at $\gamma = 1.6$, then declines. On the one hand, the supply of informal financing increases with γ because of the relaxation of the constraint on informal financing. In addition, the implicit cost of borrowing informal loans, which is the marginal product of capital of the informal lender, is also lower, since the informal lender becomes less financially constrained. On the other hand, the demand for informal financing decreases with γ because en-

trepreneurs exhaust their formal credit capacity before turning to informal loans.[13] As the formal financial market develops, more entrepreneurs' financing needs can be met by the formal financial market, therefore the demand for informal loans declines.

In summary, when γ increases, the supply and demand of informal financing move in opposite directions. As shown in Figure 7, at the early stages of economic development, the supply force dominates, while in the later stages of economic development, the demand force dominates.

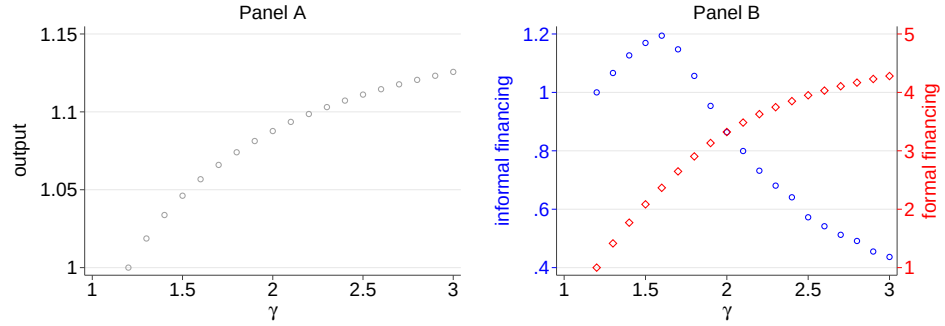

Figure 7: **The aggregate dynamics of financial development**

Notes: These two figures plot the dynamics of output and the size of informal and formal financing generated by the model. The values are normalized to 1 when $\gamma = 1.1$.

5.3 Quantifying the gain from informal financing

In this section, we quantify the gains from informal financing in five countries at different stages of economic development.[14] We recalibrate our benchmark model to match four other countries besides Mexico: Argentina, Colombia, Nigeria and Kenya. For each country, we recalibrate three key parameters—aggregate TFP A,

[13]The pecking order is by assumption here. Alternatively, we could generate the pecking order in equilibrium by assuming that informal financing incurs a monitoring cost.

[14]All five countries have more than 500 firm-level observations in the WBES database.

formal financing collateral constraint γ, and the informal financing friction ϵ—to match their country-specific targets while keeping the other parameters fixed at their benchmark levels (see Table 2).

Table 3: **Country-specific calibration**

Country	A	Data	Model	γ	Data	Model	ϵ	Data	Model
Mexico	1	1	1	1.80	0.41	0.41	0.39	0.20	0.20
Argentina	0.96	0.87	0.87	1.63	0.32	0.32	0.33	0.15	0.15
Colombia	0.65	0.55	0.55	1.68	0.39	0.39	0.42	0.21	0.21
Nigeria	0.24	0.13	0.13	1.32	0.24	0.24	0.33	0.13	0.13
Kenya	0.18	0.10	0.10	1.55	0.37	0.37	0.22	0.11	0.11

Notes: This table summarizes the calibration results of five countries (Mexico, Brazil, Colombia, Nigeria and Kenya). The TFP and output of Mexico are normalized to 1. TFP for the other countries are calibrated to match output per capita as a share of Mexico.

Table 3 shows that the country-specific calibrations match these three aggregate data moments. On average, the middle-income countries—Mexico, Argentina and Colombia—have more efficient formal financial markets (higher γ) and lower frictions in informal financing (high ϵ) than the low-income countries—Nigeria and Kenya.

Figure 8 shows that the calibration also delivers a distribution of informal financing and employment share across firm size that matches the data. In all four countries, the top quartile of firms contributes to at least 60 percent of total employment. In addition, the usage of informal financing generally increases with firm size, similar to the pattern of Mexican firms documented in Figure 6.

With the calibrated models, we compute the gain from informal financing for each country as the percent difference between the output of the benchmark economy and a counterfactual economy without informal financing. In the counterfactual economy, we set $\epsilon = 0$ and keep all the other parameters at their benchmark levels.

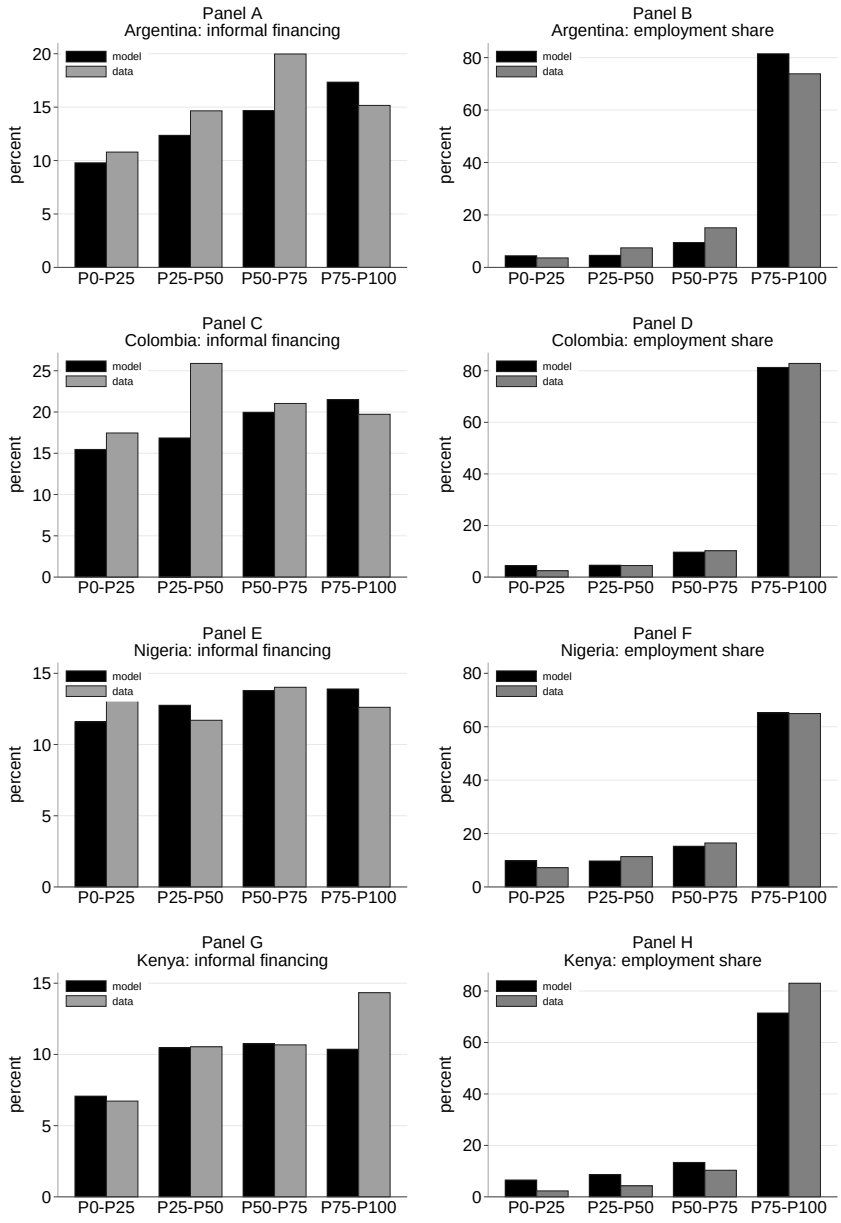

Figure 8: **Informal financing and employment across firm size by country (non-targeted moments)**

Source: WBES (2009-2014), model output.
Notes: These figures plot the distribution of informal financing and employment share by firm size (from smallest quartile to the largest quartile) in each country. In panel A, informal financing is computed as the average of the share of informal financing in fixed asset investment and working capital in WBES data, and it is computed as informal financing as a share of output in the model.

We find that for all five countries, the output of the counterfactual economy is lower than the benchmark economy, which reflects gains from informal financing through better allocation of resources across entrepreneurs. Table 4 shows the gains from informal financing range from 3.15 percent (Kenya) to 6.02 percent (Colombia) for the five countries. In general, the relatively richer countries gain more from informal financing than the poor countries.

Table 4: **Output gain from informal financing by country**

Country	GDP pc (Mexico=1)	Gain (percent)
Mexico	1	5.70
Argentina	0.869	4.82
Colombia	0.554	6.02
Nigeria	0.131	4.30
Kenya	0.100	3.15

Notes: This table displays GDP per capita (normalized, Mexico=1) of the five countries and the gain from informal financing in the model.

5.4 Gain from financial development

Figure 9 displays the changes in aggregate output and the volume of formal and informal financing when Argentina, Colombia, Nigeria and Kenya are endowed with Mexico's formal financial market, that is, we set $\gamma = 1.8$ for these countries. In all four countries, the more efficient formal financial market leads to more formal financing and higher output, and notably, more informal financing as well. In Kenya, the relative increase in informal financing (18 percent) is even higher than that of formal financing (11 percent). These results suggest that the development of the formal financial market also enhances the role of informal financing in allocating resources across heterogeneous entrepreneurs.

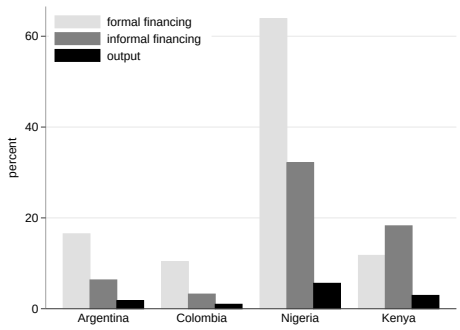

Figure 9: **Gain from an improvement in formal financial market**

Notes: This figure plots the gains in output, formal and informal financing from an improvement in the formal financial market to the level of the Mexican economy ($\gamma = 1.8$).

6 Conclusion

This paper provides a cross-country analysis of informal financing to shed light on its role in the process of economic development. Contrary to traditional views, we find that rich countries—in our sample, they are the middle-income countries—benefit more from informal financing than the poorest countries. More broadly speaking, the goal of this paper is to reach a more comprehensive understanding of financial development and its relationship with economic growth by studying the interactions among different types of financial activities. We emphasize the substitution between informal and formal financing at the firm level and how the substitutability varies with aggregate economic conditions such as the development of formal financial market.

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Appendix

A Additional figures and tables

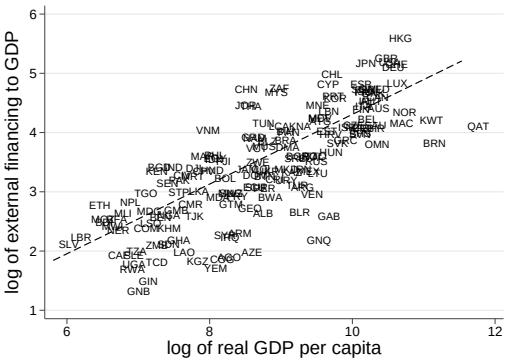

Figure A1: **Cross-country income differences and financial development**

Notes: This figure shows the cross-country correlation between the logarithm of GDP per capita (x axis) and the development of financial market (y axis). The level of financial market development is measured by the ratio of external financing to GDP, which is computed using the Financial Development and Structure Dataset (see Čihák et al., 2012) following the definition in Buera, Kaboski and Shin (2011). GDP per capita is computed using data from the Penn World Table 8.0.

Table A1: **Summary statistics of WBES data by country**

Country name	Country code	N	size			sector			inf^{fa}							inf^{wc}			
			small (percent)	median (percent)	large (percent)	manufacturing (percent)	service (percent)	others (percent)	min (percent)	max (percent)	mean (percent)	median (percent)	s.d.	min (percent)	max (percent)	mean (percent)	median (percent)	s.d.	constrained (percent)
Afghanistan	AFG	303	46.2	42.2	11.6	33.3	11.2	55.4	0	50	0.9	0	5.8	0	100	8.5	0	20.1	48.8
Albania	ALB	294	59.2	35.4	5.4	40.8	25.5	33.7	0	70	1.4	0	7.6	0	100	4.1	0	13.8	18.7
Angola	AGO	190	69.5	26.3	4.2	54.7	10.5	34.7	0	50	0.8	0	5.5	0	100	10.4	0	21.2	61.6
Argentina	ARG	965	35.8	40.5	23.7	67.4	16.1	16.6	0	100	8.4	0	23.1	0	100	23.5	10	29.1	37.1
Bangladesh	BGD	996	23.0	30.9	46.1	83.9	2.9	13.2	0	100	0.5	0	5.0	0	100	6.6	0	16.4	28.4
Belarus	BLR	128	37.5	33.6	28.9	42.2	18.0	39.8	0	100	3.1	0	12.8	0	100	6.0	0	16.4	15.6
Bhutan	BTN	177	40.1	41.2	18.6	17.5	4.0	32.2	0	50	1.4	0	6.9	0	55	5.8	0	12.9	21.5
Bolivia	BOL	392	37.0	43.6	19.4	53.1	18.6	28.3	0	100	5.1	0	16.8	0	100	14.0	0	23.4	25.3
Bosnia and Herzegovina	BIH	175	48.6	37.1	14.3	35.4	26.3	38.3	0	100	9.0	0	22.5	0	100	22.7	0	31.0	14.3
Botswana	BWA	253	56.9	28.9	14.2	33.6	28.9	37.5	0	100	3.9	0	14.2	0	100	22.8	10	28.1	37.5
Brazil	BRA	974	34.5	47.3	18.2	74.5	9.5	15.9	0	100	14.6	0	30.3	0	100	22.1	0	31.3	50.9
Bulgaria	BGR	735	35.5	39.0	25.4	49.3	28.2	22.6	0	100	2.7	0	13.2	0	100	5.2	0	15.5	19.2
Burkina Faso	BFA	124	56.5	33.9	9.7	17.7	26.6	55.6	0	100	2.7	0	13.9	0	100	10.2	0	22.4	68.5
Burundi	BDI	192	68.2	28.1	3.6	38.5	17.7	43.8	0	100	1.2	0	8.4	0	70	10.0	0	14.6	43.8
Cameroon	CMR	115	40.9	42.6	16.5	35.7	24.3	40.0	0	100	11.3	0	23.2	0	100	22.9	10	26.0	52.2
Chile	CHL	1034	27.5	45.4	27.2	72.4	15.8	11.8	0	100	8.7	0	24.6	0	100	20.2	0	28.1	20.4
China	CHN	1103	13.2	41.3	45.5	71.6	2.8	25.6	0	100	0.9	0	6.2	0	100	3.2	0	10.4	5.7
Colombia	COL	1057	34.7	38.9	26.4	70.4	16.5	13.2	0	100	9.8	0	25.1	0	100	27.7	20	30.0	21.9
Congo, Dem. Rep.	COD	417	75.8	20.6	3.6	42.4	19.9	37.6	0	100	3.3	0	12.1	0	100	11.3	0	18.6	58.5
Costa Rica	CRI	246	35.4	43.9	20.7	61.8	19.1	19.1	0	100	3.1	0	13.4	0	100	10.9	0	22.3	39.0
Cote d'Ivoire	CIV	149	66.4	18.8	14.8	28.9	16.1	55.0	0	100	5.0	0	17.7	0	100	7.7	0	22.4	71.1
Croatia	HRV	712	43.7	35.4	20.9	50.1	19.9	29.9	0	100	5.5	0	18.6	0	100	18.2	0	29.1	20.8
Czech Republic	CZE	118	52.5	37.3	10.2	43.2	19.5	37.3	0	100	4.2	0	16.4	0	92	8.3	0	19.9	16.9
Dominican Republic	DOM	152	24.3	42.1	33.6	40.1	27.6	32.2	0	100	10.2	0	25.6	0	95	31.6	30	22.7	17.1
Ecuador	ECU	565	33.5	42.7	23.9	49.2	26.2	24.6	0	100	14.3	0	31.4	0	100	25.5	8	32.3	23.4
Egypt	EGY	322	39.1	35.4	25.5	79.2	3.7	17.1	0	100	3.9	0	15.9	0	100	11.0	0	22.2	37.9
El Salvador	SLV	477	28.7	38.8	32.5	57.4	16.8	25.8	0	100	10.8	0	28.3	0	100	17.7	0	26.2	24.9
Estonia	EST	125	54.4	32.8	12.8	32.8	28.0	39.2	0	70	2.8	0	10.9	0	100	20.5	0	28.2	4.8
Ethiopia	ETH	157	51.6	26.8	21.7	41.4	19.1	39.5	0	20	0.1	0	1.6	0	91	2.3	0	10.6	45.9
Georgia	GEO	130	52.3	32.3	15.4	40.8	27.7	31.5	0	100	2.3	0	12.3	0	100	6.7	0	19.2	19.2
Ghana	GHA	562	63.7	27.9	8.4	55.3	20.6	24.0	0	100	3.6	0	12.5	0	100	16.8	10	20.4	63.2
Guatemala	GTM	488	25.2	36.3	38.5	63.7	15.6	20.7	0	100	10.6	0	26.4	0	100	21.4	0	30.2	19.1
Guinea	GIN	122	87.7	8.2	4.1	59.8	18.0	22.1	0	60	2.6	0	11.1	0	100	22.9	17.5	25.2	56.6
Honduras	HND	297	35.4	34.3	30.3	60.6	19.9	19.5	0	100	4.0	0	16.4	0	100	11.6	0	24.7	19.2
India	IND	1660	24.2	51.6	24.2	78.4	8.3	13.4	0	100	0.2	0	3.4	0	100	4.6	0	13.3	12.1
Indonesia	IDN	312	45.5	29.2	25.3	82.7	7.1	10.3	0	70	0.9	0	6.3	0	100	7.0	0	22.0	11.9
Iraq	IRQ	265	72.1	25.7	2.3	64.9	35.1	0.0	0	100	4.9	0	12.2	0	100	10.1	0	18.2	45.7
Israel	ISR	131	34.4	40.5	25.2	50.4	15.3	34.4	0	92	1.5	0	10.7	0	100	8.2	0	23.1	4.6
Jamaica	JAM	110	30.9	42.7	26.4	48.2	21.8	30.0	0	50	2.0	0	8.3	0	100	24.5	20	20.4	35.5
Kazakhstan	KAZ	157	44.6	38.9	16.6	36.9	31.2	31.8	0	70	1.9	0	8.9	0	100	7.9	0	21.8	12.7
Kenya	KEN	571	38.7	34.5	26.8	59.7	17.7	22.6	0	100	3.7	0	14.2	0	100	22.0	15	23.8	33.3
Laos	LAO	197	45.2	37.1	17.8	17.8	6.6	29.9	0	70	2.2	0	10.5	0	90	4.2	0	14.3	13.7
Latvia	LVA	110	60.0	25.5	14.5	40.0	24.5	35.5	0	100	7.8	0	23.8	0	100	17.4	0	29.7	13.6
Lithuania	LTU	111	45.9	36.0	18.0	37.8	27.0	35.1	0	100	4.9	0	17.2	0	100	25.3	10	33.0	13.5
Macedonia	MKD	169	60.9	31.4	7.7	34.9	24.3	40.8	0	100	2.4	0	12.6	0	100	6.1	0	17.0	18.9
Madagascar	MDG	263	39.5	40.3	20.2	47.1	20.9	31.9	0	100	7.8	0	23.5	0	100	15.9	0	27.5	29.3
Malawi	MWI	164	40.2	34.1	25.6	18.9	16.5	23.2	0	100	3.8	0	12.4	0	85	14.6	0	23.1	40.9
Mali	MLI	256	81.3	16.4	2.3	55.9	23.8	20.3	0	100	2.3	0	10.2	0	100	12.4	10	17.0	59.8
Mauritania	MRT	127	64.6	27.6	7.9	26.0	12.6	61.4	0	100	4.4	0	16.9	0	100	15.2	0	22.6	53.5
Mauritius	MUS	185	44.3	37.8	17.8	38.4	28.1	33.5	0	100	2.7	0	14.6	0	100	6.4	0	18.5	41.6
Mexico	MEX	893	35.9	31.9	32.1	77.5	10.2	12.3	0	100	14.8	0	31.0	0	100	23.5	0	32.4	19.9

Country name	Country code	N	size			sector					inf^{fa}						inf^{wc}		
			small (percent)	median (percent)	large (percent)	manufacturing (percent)	service (percent)	others (percent)	min (percent)	max (percent)	mean (percent)	median (percent)	s.d.	min (percent)	max (percent)	mean (percent)	median (percent)	s.d.	constrained (percent)
Mongolia	MNG	188	43.1	44.7	12.2	35.1	33.0	31.9	0	100	2.2	0	12.1	0	100	11.3	0	23.1	24.5
Morocco	MAR	126	24.6	43.7	31.7	50.8	25.4	23.8	0	100	5.4	0	18.4	0	100	15.8	0	24.9	26.2
Mozambique	MOZ	140	60.0	27.9	12.1	73.6	19.3	7.1	0	100	7.6	0	20.3	0	70	18.4	15	19.3	53.6
Myanmar	MMR	201	59.2	25.9	14.9	55.7	12.9	31.3	0	100	1.4	0	9.5	0	100	6.9	0	19.9	21.9
Namibia	NAM	294	74.5	21.1	4.4	29.9	25.9	44.2	0	100	0.9	0	7.9	0	100	12.8	0	23.5	36.1
Nepal	NPL	268	45.9	41.0	13.1	45.9	17.2	36.9	0	70	0.8	0	6.9	0	70	4.2	0	12.8	19.4
Nicaragua	NIC	324	40.7	42.3	17.0	63.3	14.8	21.9	0	100	2.8	0	12.5	0	100	11.8	0	24.0	20.4
Nigeria	NGA	1671	71.5	24.2	4.3	47.8	20.9	31.3	0	100	3.3	0	10.3	0	100	25.0	20	27.2	16.5
Pakistan	PAK	149	25.5	42.3	32.2	78.5	8.1	13.4	0	80	3.0	0	10.8	0	90	6.2	0	15.0	22.1
Panama	PAN	300	50.0	34.0	16.0	44.7	33.0	22.3	0	100	5.3	0	20.2	0	100	7.4	0	18.4	10.7
Paraguay	PRY	510	35.9	43.9	20.2	51.0	22.4	26.7	0	100	10.0	0	25.6	0	100	14.6	0	24.9	25.7
Peru	PER	906	28.6	43.5	27.9	72.3	12.3	15.5	0	100	8.8	0	24.8	0	100	23.7	0	30.0	14.3
Philippines	PHL	382	18.8	47.6	33.5	74.6	9.9	15.4	0	100	7.2	0	22.1	0	100	15.1	0	27.0	13.4
Poland	POL	162	49.4	34.0	16.7	36.4	27.8	35.8	0	100	9.3	0	21.3	0	100	20.2	0	26.8	17.3
Romania	ROU	244	47.5	33.6	18.9	35.2	23.4	41.4	0	100	3.2	0	13.8	0	100	17.4	0	27.2	32.0
Russia	RUS	1313	40.5	41.1	18.4	38.2	8.1	53.6	0	100	3.6	0	14.5	0	100	10.0	0	22.6	26.0
Rwanda	RWA	221	61.1	25.3	13.6	11.3	10.0	34.8	0	100	3.2	0	13.0	0	100	11.7	0	20.1	27.6
Senegal	SEN	217	68.2	24.0	7.8	59.4	15.2	25.3	0	100	6.9	0	22.6	0	100	15.2	4	24.6	47.9
Serbia	SRB	164	53.0	31.1	15.9	31.7	34.1	34.1	0	100	12.6	0	29.0	0	100	24.4	0	34.2	14.6
Slovenia	SVN	179	49.7	30.7	19.6	34.6	30.2	35.2	0	90	2.8	0	13.3	0	100	3.7	0	15.6	26.3
South Africa	ZAF	284	28.2	50.0	21.8	80.6	10.2	9.2	0	100	6.2	0	20.6	0	100	25.3	20	20.7	14.8
Sri Lanka	LKA	118	40.7	30.5	28.8	64.4	13.6	22.0	0	100	2.6	0	12.2	0	100	13.8	0	24.0	28.0
Sudan	SDN	300	68.7	26.3	5.0	16.0	33.3	50.7	0	100	6.8	0	15.9	0	100	14.9	0	23.5	35.7
Swaziland	SWZ	103	69.9	20.4	9.7	26.2	34.0	39.8	0	100	6.1	0	20.8	0	100	23.1	15	27.5	36.9
Sweden	SWE	233	43.3	45.5	11.2	68.2	9.9	21.9	0	100	2.4	0	14.4	0	100	9.0	0	20.4	4.7
Tajikistan	TJK	115	47.8	42.6	9.6	34.8	30.4	34.8	0	80	1.4	0	8.4	0	100	8.0	0	21.0	27.0
Tanzania	TZA	369	56.9	33.9	9.2	68.3	11.1	20.6	0	50	0.9	0	5.3	0	100	18.0	10	21.8	51.5
Trinidad & Tobago	TTO	112	44.6	27.7	27.7	25.0	31.3	43.8	0	100	5.9	0	13.4	0	100	20.7	15	18.1	26.8
Tunisia	TUN	245	26.5	40.8	32.7	60.8	7.8	31.4	0	100	2.9	0	12.8	0	100	18.9	10	23.7	21.2
Turkey	TUR	331	39.9	34.4	25.7	81.0	7.6	11.5	0	90	2.1	0	8.6	0	100	9.6	0	19.9	8.2
Uganda	UGA	376	59.0	31.4	9.6	63.8	14.9	21.3	0	70	2.1	0	8.5	0	100	15.5	10	19.4	44.7
Ukraine	UKR	154	35.1	42.9	22.1	86.4	0.0	13.6	0	100	6.7	0	18.9	0	100	13.8	0	23.2	25.3
Uruguay	URY	578	32.9	43.1	24.0	62.3	13.1	24.6	0	100	5.8	0	19.4	0	100	19.7	0	27.8	20.1
Uzbekistan	UZB	109	33.0	33.0	33.9	44.0	22.0	33.9	0	100	0.9	0	9.6	0	25	0.4	0	2.8	9.2
Venezuela	VEN	210	51.0	32.9	16.2	17.6	7.1	10.0	0	100	7.6	0	24.2	0	100	10.3	0	23.8	19.0
Vietnam	VNM	609	17.1	45.0	37.9	76.0	10.0	14.0	0	100	1.6	0	10.4	0	100	11.2	0	22.0	16.7
Yemen	YEM	128	62.5	27.3	10.2	49.2	20.3	30.5	0	100	10.4	0	28.2	0	100	15.5	0	28.7	38.3
Zambia	ZMB	372	48.7	37.9	13.4	65.9	15.6	18.5	0	100	4.2	0	16.2	0	100	15.6	0	23.6	32.8
Zimbabwe	ZWE	111	29.7	42.3	27.9	61.3	13.5	25.2	0	90	5.6	0	16.8	0	100	11.8	0	22.8	69.4
Yemen	YEM	128	62.5	27.3	10.2	49.2	20.3	30.5	0	100	10.4	0	28.2	0	100	15.5	0	28.7	38.3
Zambia	ZMB	372	48.7	37.9	13.4	65.9	15.6	18.5	0	100	4.2	0	16.2	0	100	15.6	0	23.6	32.8
Zimbabwe	ZWE	111	29.7	42.3	27.9	61.3	13.5	25.2	0	90	5.6	0	16.8	0	100	11.8	0	22.8	69.4

Notes: According to the WBES definition, small firms are those that employ more than 5 and fewer than 19 workers, median firms are those that employ more than 19 and fewer than 100 workers, and large firms are those that employ more than 100 workers. inf^{fa} is the share of informal finance in fixed asset investment, while inf^{wc} is the share of informal finance in working capital.

B Proofs

B.1 Without informal financing

Consider an entrepreneur household (a, z_1, z_2). Without the chance of engaging in informal financing, the two members of the household make production decisions separately and their optimization problems read

$$\begin{aligned}\max_{k_1,l_1} \quad & Az_1k_1^{\alpha}l_1^{\chi} - (r+\delta)k_1 - wl_1 \; s.t. \; k_1 \leq \gamma a_1, \\ \max_{k_2,l_2} \quad & Az_2k_2^{\alpha}l_2^{\chi} - (r+\delta)k_2 - wl_2 \; s.t. \; k_2 \leq \gamma a_2.\end{aligned}$$

The unconstrained solution to the above problem is

$$\begin{aligned}k_1 &= [Az_1(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}, \\ l_1 &= [Az_1(\frac{\alpha}{r+\delta})^{\alpha}(\frac{\chi}{w})^{1-\alpha}]^{\frac{1}{1-\alpha-\chi}}, \\ k_2 &= [Az_2(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}, \\ l_2 &= [Az_2(\frac{\alpha}{r+\delta})^{\alpha}(\frac{\chi}{w})^{1-\alpha}]^{\frac{1}{1-\alpha-\chi}}.\end{aligned}$$

The unconstrained profits are

$$\begin{aligned}\pi_1(a_1, z_1) &= (Az_1)^{\frac{1}{1-\alpha-\chi}}(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}}(1-\alpha-\chi), \\ \pi_2(a_2, z_2) &= (Az_2)^{\frac{1}{1-\alpha-\chi}}(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}}(1-\alpha-\chi), \\ \pi(a, z_1, z_2) &= [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_2)^{\frac{1}{1-\alpha-\chi}}](\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}}(1-\alpha-\chi).\end{aligned}$$

Next consider the case where $a_1 = a_2 = \frac{1}{2}a$ and $z_2 \geq z_1$. The solution to the entrepreneurs' problem can be analyzed in the following three cases.

Case 1 If $\frac{1}{2}\gamma a \geq [Az_2(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}$, it holds that $\frac{1}{2}\gamma a \geq [Az_1(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}$ because $z_2 \geq z_1$. In this case, both entrepreneurs are unconstrained; therefore

$$\begin{aligned}\pi(a, z_1, z_2) &= [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_2)^{\frac{1}{1-\alpha-\chi}}](\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}}(1-\alpha-\chi), \\ \pi_a(a, z_1, z_2) &= 0.\end{aligned}$$

Case 2 If $\frac{1}{2}\gamma a \geq [Az_1(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}$ and $\frac{1}{2}\gamma a < [Az_2(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}$, in this case entrepreneur z_1 achieved unconstrained production scale, whereas entrepreneur

z_2 is constrained, such that

$$\begin{aligned}
\pi(a,z_1,z_2) &= (Az_1)^{\frac{1}{1-\alpha-\chi}}(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}}(1-\alpha-\chi) \\
&\quad +Az_2k_2^{\alpha}(\frac{\chi Az_2k_2^{\alpha}}{w})^{\frac{\chi}{1-\chi}}-(r+\delta)k_2-w(\frac{\chi Az_2k_2^{\alpha}}{w})^{\frac{1}{1-\chi}} \\
&= (Az_1)^{\frac{1}{1-\alpha-\chi}}(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}}(1-\alpha-\chi) \\
&\quad +(Az_2)^{\frac{1}{1-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\chi}}k_2^{\frac{\alpha}{1-\chi}}-(Az_2)^{\frac{1}{1-\chi}}(\frac{\chi}{w^{\chi}})^{\frac{1}{1-\chi}}k_2^{\frac{\alpha}{1-\chi}} \\
&\quad -(r+\delta)k_2 \\
&= (Az_1)^{\frac{1}{1-\alpha-\chi}}(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}}(1-\alpha-\chi) \\
&\quad +(Az_2)^{\frac{1}{1-\chi}}[(\frac{\chi}{w})^{\frac{\chi}{1-\chi}}-(\frac{\chi}{w^{\chi}})^{\frac{1}{1-\chi}}](\frac{1}{2}\gamma a)^{\frac{\alpha}{1-\chi}} \\
&\quad -(r+\delta)\frac{1}{2}\gamma a. \\
\pi_a(a,z_1,z_2) &= \frac{\alpha}{1-\chi}(Az_2)^{\frac{1}{1-\chi}}[(\frac{\chi}{w})^{\frac{\chi}{1-\chi}}-(\frac{\chi}{w^{\chi}})^{\frac{1}{1-\chi}}](\frac{1}{2}\gamma)^{\frac{\alpha}{1-\chi}}a^{\frac{\alpha+\chi-1}{1-\chi}} \\
&\quad -(r+\delta)\frac{1}{2}\gamma.
\end{aligned}$$

Case 3 If $\frac{1}{2}\gamma a < [Az_1(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}$, both entrepreneurs are constrained; therefore, the profit functions and the gradient of the profit function read

$$\begin{aligned}
\pi(a,z_1,z_2) &= [(Az_1)^{\frac{1}{1-\chi}}+(Az_2)^{\frac{1}{1-\chi}}][(\frac{\chi}{w})^{\frac{\chi}{1-\chi}}-(\frac{\chi}{w^{\chi}})^{\frac{1}{1-\chi}}](\frac{1}{2}\gamma a)^{\frac{\alpha}{1-\chi}}-(r+\delta)\gamma a, \\
\pi_a(a,z_1,z_2) &= \frac{\alpha}{1-\chi}[(Az_1)^{\frac{1}{1-\chi}}+(Az_2)^{\frac{1}{1-\chi}}][(\frac{\chi}{w})^{\frac{\chi}{1-\chi}}-(\frac{\chi}{w^{\chi}})^{\frac{1}{1-\chi}}](\frac{1}{2}\gamma)^{\frac{\alpha}{1-\chi}}a^{\frac{\alpha+\chi-1}{1-\chi}}-(r+\delta)\gamma.
\end{aligned}$$

B.2 With informal financing

The optimization problem of the individuals can be written as

$$\begin{aligned}
\pi(a,z_1,z_2) &= \max\ Az_1k_1^{\alpha}l_1^{\chi}+Az_2k_2^{\alpha}l_2^{\chi}-(r+\delta)(k_1+k_2)-w(l_1+l_2) \\
s.t. &\quad k_1+k_2\leq\gamma(a_1+a_2).
\end{aligned}$$

The unconstrained solutions to the above problem are

$$\begin{aligned}
k_1 &= [Az_1(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}, \\
l_1 &= [Az_1(\frac{\alpha}{r+\delta})^{\alpha}(\frac{\chi}{w})^{1-\alpha}]^{\frac{1}{1-\alpha-\chi}}, \\
k_2 &= [Az_2(\frac{\alpha}{r+\delta})^{1-\chi}(\frac{\chi}{w})^{\chi}]^{\frac{1}{1-\alpha-\chi}}, \\
l_2 &= [Az_2(\frac{\alpha}{r+\delta})^{\alpha}(\frac{\chi}{w})^{1-\alpha}]^{\frac{1}{1-\alpha-\chi}}.
\end{aligned}$$

It follows that the profit function of the unconstrained solution can be written as

$$\begin{aligned}
\pi(a, z_1, z_2) &= Az_1k_1^{\alpha}l_1^{\chi} + Az_2k_2^{\alpha}l_2^{\chi} - (r+\delta)(k_1+k_2) - w(l_1+l_2) \\
&= [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_1)^{\frac{1}{1-\alpha-\chi}}](\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} \\
&\quad -(r+\delta)[(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_1)^{\frac{1}{1-\alpha-\chi}}](\frac{\alpha}{r+\delta})^{\frac{1-\chi}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} \\
&\quad -w[(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_1)^{\frac{1}{1-\alpha-\chi}}](\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{1-\alpha}{1-\alpha-\chi}} \\
&= [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_1)^{\frac{1}{1-\alpha-\chi}}] \\
&\quad [(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} - (\frac{\alpha^{1-\chi}}{(r+\delta)^{\alpha}})^{\frac{1}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}} \\
&\quad -(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi^{1-\alpha}}{w^{\chi}})^{\frac{1}{1-\alpha-\chi}}] \\
&= [(Az_1)^{\frac{1}{1-\alpha-\chi}} + (Az_1)^{\frac{1}{1-\alpha-\chi}}](\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\alpha-\chi}}(1-\alpha-\chi).
\end{aligned}$$

The FOCs of the constrained solution can be written as

$$\begin{aligned}
k_1: \ A\alpha z_1k_1^{\alpha-1}l_1^{\chi} &= r+\delta+\mu, \\
k_2: \ A\alpha z_2k_2^{\alpha-1}l_2^{\chi} &= r+\delta+\mu, \\
l_1: \ A\chi z_1k_1^{\alpha}l_1^{\chi-1} &= w, \\
l_2: \ A\chi z_2k_2^{\alpha}l_2^{\chi-1} &= w.
\end{aligned}$$

Rewrite FOCs w.r.t. l_1 and l_2 as $l_1 = (\frac{\chi Az_1k_1^{\alpha}}{w})^{\frac{1}{1-\chi}}$ and $l_2 = (\frac{\chi Az_2k_2^{\alpha}}{w})^{\frac{1}{1-\chi}}$. Take them back to the FOCs w.r.t. to k_1 and k_2, and we have

$$\begin{aligned}
k_1^{\frac{1-\alpha-\chi}{1-\chi}} &= (Az_1)^{\frac{1}{1-\chi}}(\frac{\alpha}{r+\delta+\mu})(\frac{\chi}{w})^{\frac{\chi}{1-\chi}}, \\
k_2^{\frac{1-\alpha-\chi}{1-\chi}} &= (Az_2)^{\frac{1}{1-\chi}}(\frac{\alpha}{r+\delta+\mu})(\frac{\chi}{w})^{\frac{\chi}{1-\chi}}.
\end{aligned}$$

The above two equations give the capital ratio as $\frac{k_1}{k_2} = (\frac{z_1}{z_2})^{\frac{1}{1-\alpha-\chi}}$. Since in this

case the constraint $k_1 + k_2 \leq \gamma(a_1 + a_2)$, we can compute

$$\begin{aligned} k_1 &= \frac{\hat{z}}{1+\hat{z}}\gamma(a_1+a_2), \\ k_2 &= \frac{1}{1+\hat{z}}\gamma(a_1+a_2), \end{aligned}$$

where $\hat{z} = (\frac{z_1}{z_2})^{\frac{1}{1-\alpha-\chi}}$, it still holds that $l_1 = (\frac{\chi A z_1 k_1^\alpha}{w})^{\frac{1}{1-\chi}}$ and $l_2 = (\frac{\chi A z_2 k_2^\alpha}{w})^{\frac{1}{1-\chi}}$. We can then compute the profit function with constraint as

$$\begin{aligned} \pi(a, z_1, z_2) &= Az_1 k_1^\alpha l_1^\chi + Az_2 k_2^\alpha l_2^\chi - (r+\delta)(k_1+k_2) - w(l_1+l_2) \\ &= Az_1 k_1^\alpha (\frac{\chi A z_1 k_1^\alpha}{w})^{\frac{\chi}{1-\chi}} + Az_2 k_2^\alpha (\frac{\chi A z_2 k_2^\alpha}{w})^{\frac{\chi}{1-\chi}} \\ &\quad -(r+\delta)(k_1+k_2) \\ &\quad -w(\frac{\chi A z_1 k_1^\alpha}{w})^{\frac{1}{1-\chi}} - w(\frac{\chi A z_2 k_2^\alpha}{w})^{\frac{1}{1-\chi}} \\ &= (Az_1)^{\frac{1}{1-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\chi}} k_1^{\frac{\alpha}{1-\chi}} + (Az_2)^{\frac{1}{1-\chi}}(\frac{\chi}{w})^{\frac{\chi}{1-\chi}} k_2^{\frac{\alpha}{1-\chi}} \\ &\quad -(r+\delta)(k_1+k_2) \\ &\quad -(Az_1)^{\frac{1}{1-\chi}}(\frac{\chi}{w^\chi})^{\frac{1}{1-\chi}} k_1^{\frac{\alpha}{1-\chi}} - (Az_2)^{\frac{1}{1-\chi}}(\frac{\chi}{w^\chi})^{\frac{1}{1-\chi}} k_2^{\frac{\alpha}{1-\chi}} \\ &= [(Az_1)^{\frac{1}{1-\chi}}((\frac{\chi}{w})^{\frac{\chi}{1-\chi}} - (\frac{\chi}{w^\chi})^{\frac{1}{1-\chi}})] k_1^{\frac{\alpha}{1-\chi}} \\ &\quad +[(Az_2)^{\frac{1}{1-\chi}}((\frac{\chi}{w})^{\frac{\chi}{1-\chi}} - (\frac{\chi}{w^\chi})^{\frac{1}{1-\chi}})] k_2^{\frac{\alpha}{1-\chi}} \\ &\quad -(r+\delta)(k_1+k_2). \end{aligned}$$

Take the equations with k_1 and k_2 back to the above equations, and we get

$$\begin{aligned} \pi(a, z_1, z_2) &= [(Az_1)^{\frac{1}{1-\chi}}(\frac{\hat{z}}{1+\hat{z}})^{\frac{\alpha}{1-\chi}} + (Az_2)^{\frac{1}{1-\chi}}(\frac{1}{1+\hat{z}})^{\frac{\alpha}{1-\chi}}] \\ &\quad [(\frac{\chi}{w})^{\frac{\chi}{1-\chi}} - (\frac{\chi}{w^\chi})^{\frac{1}{1-\chi}}]\gamma^{\frac{\alpha}{1-\chi}} a^{\frac{\alpha}{1-\chi}} - (r+\delta)\gamma a \end{aligned}$$

Denote $\mathbb{B} = [(Az_1)^{\frac{1}{1-\chi}}(\frac{\hat{z}}{1+\hat{z}})^{\frac{\alpha}{1-\chi}} + (Az_2)^{\frac{1}{1-\chi}}(\frac{1}{1+\hat{z}})^{\frac{\alpha}{1-\chi}}][(\frac{\chi}{w})^{\frac{\chi}{1-\chi}} - (\frac{\chi}{w^\chi})^{\frac{1}{1-\chi}}]\gamma^{\frac{\alpha}{1-\chi}}$ and $\mathbb{C} = (r+\delta)\gamma$. Then we can write

$$\begin{aligned} \pi(a, z_1, z_2) &= \mathbb{B} a^{\frac{\alpha}{1-\chi}} - \mathbb{C} a, \\ \pi_a(a, z_1, z_2) &= \frac{\alpha}{1-\chi}\mathbb{B} a^{\frac{\alpha+\chi-1}{1-\chi}} - \mathbb{C}. \end{aligned}$$