Aggregate Fluctuations and the Role of Trade Credit*

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Abstract

This paper studies the aggregate implications of trade credit in a dynamic, general equilibrium model where heterogeneous entrepreneurs choose their lending and borrowing of trade credit in the presence of financial frictions. Motivated by empirical evidence, the model shows how trade credit flows from less constrained firms to more constrained ones, both in the cross-sectional distribution and in firms' response to heterogeneous financial shocks. In the face of an aggregate financial shock, entrepreneurs reduce their trade credit lending, further tightening their customers' borrowing constraints, resulting in an amplification of the initial shock. In contrast, when the financial shock only affects some, but not all, entrepreneurs, trade credit facilitates the flow of financing to entrepreneurs in financial distress, thereby mitigating its negative impacts. This mechanism, however, is only effective when the shock affects a sufficiently small number of entrepreneurs.

JEL codes: E23, E44, G32.
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1 Introduction

Trade credit – suppliers' lending of inputs to their customers – is an essential source of financing for U.S. firms. In 2006, for instance, total trade credit liabilities (accounts payable) of the non-financial corporate sector were approximately the same size as its monthly value-added output. Moreover, during financial turmoils, declines in trade credit often contribute significantly to the contraction of total credit. For instance, during the 2007-09 financial crisis, approximately 70% of the decrease in short-term liabilities of non-financial corporate firms was attributable to trade credit. Therefore, understanding how firms' trade credit choices are determined, both during normal times and when facing financial shocks, is crucial for understanding how a shock that originated in the financial sector propagates throughout the economy.

In this paper, I document empirical evidence supporting the existence of a financial motive behind firms' trade credit choices. Trade credit flows, in net terms, from less financially constrained firms to more financially constrained ones. Such a pattern is evident in the cross-sectional distribution and in firms' responses to heterogeneous financial shocks. The willingness of entrepreneurs to provide trade credit is sensitive to disruptions in the financial markets, which acts to amplify an aggregate financial shock. I quantify the size of this amplification effect during the 2007-09 financial crisis and show that the decrease in trade credit during the Great Recession can account for 17% of the decline in output. On the other hand, when the financial shock only hits a fraction of entrepreneurs in the economy, trade credit can mitigate its negative impact, provided that the share of affected entrepreneurs is sufficiently small.

The first part of the paper uses the Compustat dataset to establish a relationship between trade credit and firm-level financial constraints. First, in the cross-section, the net lending of trade credit is higher for larger and older firms and firms with less reliance on external financing. This pattern shows an important reallocative role played by trade credit that channels funds to financially-constrained firms. Second, the supply (demand) of trade credit decreases (increases) when firms' access to the financial markets is disrupted. I adopt

a similar strategy as Chodorow-Reich (2014) by using firms' relationship banks' exposure to Lehman Brothers as an exogenous variation in firms' access to financing in the aftermath of the Lehman bankruptcy. Firms with higher exposure to Lehman – i.e., whose relationship bank syndicated more loans with Lehman – reduced their lending and increased their borrowing of trade credit more than other firms. In other words, trade credit, in net terms, flows to firms experiencing more significant financial shocks.

Motivated by the empirical evidence, I build a quantitative model incorporating two key features: (i) firm heterogeneity in financial constraints and (ii) interaction between trade credit and firms' access to the financial market.

The model features a roundabout production process where the final good can be used as consumption, investment, or intermediate input. There is a continuum of entrepreneurs who differ by wealth and productivity, creating heterogeneity in firms' financial constraints. Trade credit and bank credit coexist, and trade credit flows – together with intermediate input goods – from relatively unconstrained to relatively constrained entrepreneurs. The process also creates collateral (accounts receivable), which the lenders of trade credit can use to obtain bank loans. Hence, trade credit redistributes credit across entrepreneurs with different degrees of financial friction and increases the overall level of credit in the economy.

Despite its minimal structure, the model can replicate both motivating facts. Specifically, it generates a cross-sectional distribution of trade credit across firm sizes consistent with empirical evidence. In addition, the model predicts trade credit flows from entrepreneurs experiencing less severe financial shocks to those experiencing more severe shocks, consistent with the pattern observed following the bankruptcy of Lehman Brothers.

I use a calibrated version of the model to evaluate the aggregate implications of trade credit quantitatively. First, I show that the presence of trade credit in the model economy significantly improves the allocative efficiency across entrepreneurs. In the steady state, aggregate productivity is 9.7% higher in the benchmark economy relative to a counterfactual economy without trade credit, as credit facilitates the redistribution of credit from less financially constrained to those with tighter financial conditions. In addition, even if I

increase bank credit in the counterfactual economy so that the aggregate debt-to-capital ratio is equal across the two models, aggregate productivity is 0.8% lower in the counterfactual economy than in the benchmark model. This suggests that trade credit performs better than bank credit in financing the most constrained entrepreneurs. Indeed, in the benchmark economy, high-productivity entrepreneurs produce a larger share of aggregate output, despite having a lower wealth share than in the counterfactual economy.

Next, I use the model to quantify the role of trade credit in a financial crisis. To generate a financial crisis of plausible magnitude, I introduce a shock process to the collateral constraints of *all* entrepreneurs, such that the model delivers the same magnitude decrease in aggregate credit market liabilities and accounts receivable upon impact as observed in the 2007–09 financial crisis. I find an amplification effect by endogenous changes in trade credit as a response to the tightening credit constraint. In particular, the fall in output following the financial shock is 17 percent larger in my benchmark economy relative to the counterfactual model without trade credit. Following the shock, there is a shift in entrepreneurs' willingness to borrow and lend trade credit, leading to an increase in the trade credit interest rate and a decrease in aggregate trade credit relative to output. The contraction in trade credit disproportionately restricts the borrowing of the most constrained entrepreneurs, who rely on trade credit for financing, while unconstrained entrepreneurs (net trade-credit lenders) gain from the rising trade credit interest rate. Both effects cause more severe misallocation and greater losses in aggregate productivity. Although the higher value of liquidity in the presence of trade credit incentives entrepreneurs to save more and hence leads to a faster accumulation of capital, the misallocation channel dominates quantitatively. Therefore, while the reallocative effect of trade credit increases the economy's steady-state output, the misallocation channel amplifies the aggregate consequences during the 2007-09 financial crisis.

While trade credit amplifies aggregate financial shocks, it can mitigate financial shocks that affect only *some*, but not all, entrepreneurs, as it allows financing to flow to entrepreneurs in idiosyncratic financial distress. The mitigation effect, however, is strongest when only a small fraction of entrepreneurs experience a tightening in their collateral constraints. As the shock becomes more widespread, fewer entrepreneurs are in a position

financially to lend trade credit, diminishing trade credit's ability to mitigate the shock. According to the model analysis, when less than 1% of entrepreneurs are affected by the financial shock, the benchmark economy suffers a smaller output loss than the counterfactual economy. Conversely, when the share of affected entrepreneurs exceeds 1%, trade credit's mitigation effect is dominated by its amplification effect, leading to greater output loss in the benchmark than in the counterfactual economy. In other words, as the financial shock spreads, affecting more and more entrepreneurs, trade credit plays an increasingly similar role as it does during an aggregate financial shock.

The findings from my quantitative analysis reconcile two empirical facts in the trade credit literature. First, the existing literature documents that trade credit plays a "redistributive" role during periods of tight credit, passing funds from more liquid firms to less liquid firms (Meltzer, 1960 and Nilsen, 2002). These papers suggest trade credit could help mitigate the impacts of financial shocks. Second, however, on the aggregate level, trade credit does not appear to increase during monetary and credit contractions (Gertler and Gilchrist, 1993). Indeed, during the Asian financial crisis and the 2007–09 global financial crisis, aggregate trade credit declined in absolute terms and relative to output (Love et al., 2007). Through the lenses of my model, these facts are consistent with how the two aspects of trade credit – its aggregate quantity and its distribution across firms – can be affected by a financial crisis. First, in a financial crisis, shocks might be distributed unevenly across firms, in which case, trade credit would flow to those facing a more severe shock. Second, the decline in aggregate trade credit reflects a contraction in overall trade credit lending activity as an endogenous response to financial market disruptions. Therefore, trade credit could impact economic outcomes through both channels, with the ultimate effect depending on the shock's aggregate and distributional properties.

There exists a long theoretical and empirical literature on trade credit (see Cuñat and Garcia-Appendini, 2012 for a review). Theoretically, the model in this paper builds on the insight that trade credit exists because suppliers have a certain comparative advantage in lending to their customers relative to financial intermediaries (Biais and Gollier, 1997, Fabbri and Menichini, 2010, Burkart and Ellingsen, 2004 and Cuñat, 2007). Empirically, evidence

documented in this paper lend supports to the existence of the financial motive behind trade credit (Schwartz, 1974 and Petersen and Rajan, 1997).

This paper also contributes to the literature on trade credit's role in propagating financial shocks (see Kiyotaki and Moore, 1997). Recent developments in this literature consider the economy's input-output structure, including the aggregate impact of trade credit in an input-output economy (Altinoglu, 2018, Luo, 2020 and Reischer, 2019) and trade credit's impact on the cross-sector correlation in economic activity (Raddatz, 2010 and Miranda-Pinto and Zhang, 2022). My paper complements this strand of the literature by exploring the role played by trade credit in the presence of firm heterogeneity. Importantly, it illustrates the importance of incorporating the underlying heterogeneity in firms' credit constraints and trade credit positions to evaluate the role of trade credit in the aggregate economy.

Lastly, I introduce trade credit into a quantitative dynamic general equilibrium model with firm heterogeneity and financial frictions, building on the existing research such as Buera and Moll (2015), Buera et al. (2015), Khan and Thomas (2013), and Zetlin-Jones and Shourideh (2017). The model extends the working capital constraint in Jermann and Quadrini (2012) by incorporating a trade credit component. Previous papers such as Zetlin-Jones and Shourideh (2017) examine how shocks originating in the financial sector affect the real economy, with calibration to match firms' net liability, a part of which is trade credit. I model trade credit separately from other liabilities (i.e., bank credit in my model). By incorporating trade credit, the model captures the endogenous response of trade credit to the exogenous shocks in the financial market, which I show is quantitatively significant. Relatedly, a recent paper by Hardy et al. (2022) incorporates trade credit in a more general model of firm-to-firm lending to study its impact on economic fluctuations in emerging market economies.

2 Empirical Motivation

This section establishes the empirical facts that motivate and discipline the model. First, I use the US Compustat data to study firms' choice of trade credit in the cross-section, focusing on the relationship between trade credit and the degree of financial constraints firms face. Following this, I study how firms' trade credit choices change following a negative financial shock by examining the events around the bankruptcy of the Lehman Brothers.

2.1 Trade credit in the cross section

Data. I construct a sample of firms using the Compustat North America database at the quarterly frequency for 2000-2007, excluding firms in the financial sector (SIC 60-69). While I focus on the 2000-2007 period, the patterns I observe are also present in more extended time series.

Trade credit shows up on both sides of a firm's balance sheet. First, accounts receivable (AR) is the amount of money due to a firm for goods or services it delivered but has not yet been paid for by the customers. It measures the firm's gross lending of trade credit to other firms and appears as a current asset. Second, accounts payable (AP) is the amount a firm owes its suppliers for goods or services it has received. It measures the gross borrowing of trade credit from their suppliers, which is a current liability. Finally, a firm's net trade credit position (net AR=AR-AP) measures the net lending (if positive) or net borrowing (if negative) of trade credit. Naturally, the size of trade credit is closely linked to firms' production scale. I normalize AR, AP, and net AR by total sales in the empirical results.

I consider three firm characteristics that are indicative of financial constraints: (i) firm age, (ii) firm size, and (iii) reliance on external financing. Existing literature such as Almeida and Campello (2007) shows that firm age and size are correlated negatively with the degree of financial constraints. Firm age is constructed using a firm's first appearance in the Compustat dataset with a non-missing closing price, and firm size is measured using their total

asset value.[1] The third measure, firms' reliance on external financing, is positively correlated with financial constraint, as argued by Rajan and Zingales (1998) and Zetlin-Jones and Shourideh (2017). External funds are more costly than internal funds due to frictions such as asymmetric information and moral hazard. Therefore, firms are more exposed to external financial frictions if their internal funds can not cover investments and thus need to raise external funds. Following Gomes (2001), I construct a measure of reliance on external financing as capital expenditures - available funds, where available funds = total cash flows - dividend payments. I then normalize external financing by the size of total fixed assets.

To limit the influence of outliers, I follow Kalemli-Ozcan et al. (2014) and winsorize the bottom and top 5 percent of the ratios I constructed (AR/sales, AP/sales, net AR/sales, and external financing/fixed assets, etc.). The final sample consists of approximately 36,000 firm-quarter observations.

Results. Figure 1 reports how trade credit varies across deciles of firm size, age, and reliance on external financing. Larger and older firms are more likely to be net trade credit lenders (net AR > 0), as shown in panels (a) and (b). For instance, while approximately 30% of the smallest decile firms are net trade credit lenders, more than 80% of the largest decile firms are. Similarly, while 65% of firms in the youngest decile are net trade credit lenders, that number rises to approximately 90% among the oldest decile. The increasing share of net trade credit lenders is driven primarily by decreasing AP/sales with firm size and age. Given the empirical relationship between age/size and financial constraint (Almeida and Campello, 2007), I interpret the patterns in panels (a) and (b) as suggestive evidence that trade credit is used by constrained firms for financing working capital.[2]

Perhaps the relationship between trade credit and firms' reliance on external financing

[1] Besides firms' first appearance in the Compustat dataset, the literature uses two alternative sources to estimate the age of public firms in the US. The first one is their IPO dates. The second one is a dataset of firm age constructed by Loughran and Ritter (2004) for a sample of public firms. These three measures are highly correlated, and the results are robust to both alternative measures.

[2] Murfin and Njoroge (2015) showed that the median net payable days declines with firm size in the Compustat dataset, except for the largest two deciles of firms. A similar, albeit less significant, pattern of average and median AP/sales can also be seen in my figure 1 for the top two size deciles. I provide a more detailed discussion of this pattern in the appendix section A.1

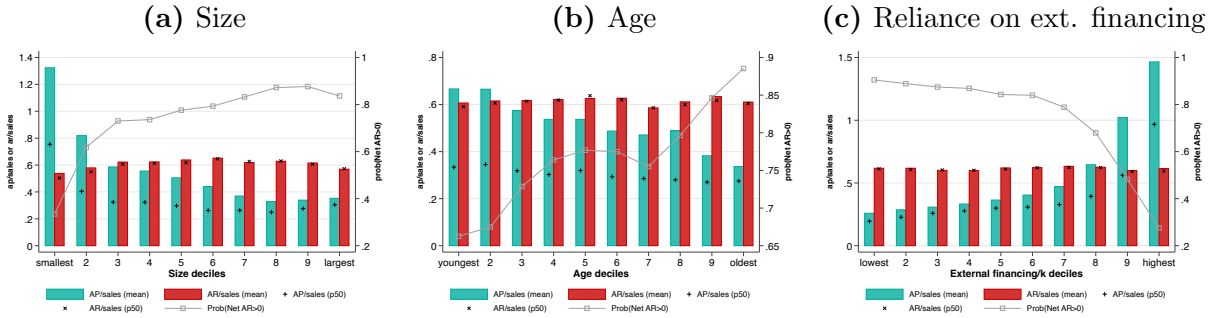


Figure 1: Trade credit by firm age, size and reliance on external financing

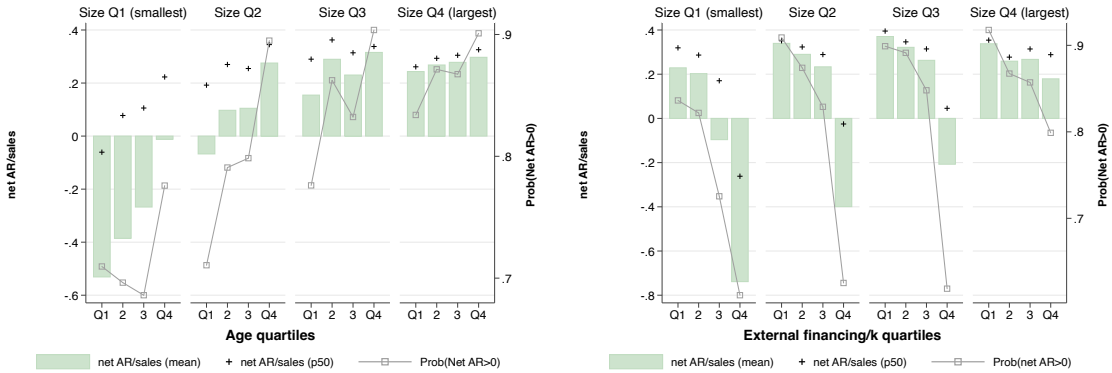
Notes: The sample includes all non-financial firms in the Compustat dataset for 2000-2007. The figures plot the choice of trade credit over firm age (left), firm size (middle) and reliance on external financing (right). The bars reflect the mean and the (+ or x) reflect the median AP/sale and AR/sales for each decile, respectively. The gray line shows the probability of being a net trade credit lender (prob(net AR>0)) in each decile.

in panel (c) provides more direct proof of this financial motive. The figure shows that firms relying more on external financing are less likely to be net trade credit lenders, a relationship difficult to explain if trade credit is driven purely by non-financial motives. Suppose, instead, that there is a financial motive behind trade credit choices. Namely, firms use trade credit to finance working capital. In that case, it is hardly surprising that firms relying more on external financing for fixed asset investment are also more likely to borrow trade credit since funds for fixed assets and working capital arguably come from a shared pool.

Figure 2 shows that these correlations survive when conditional on firm size or age. The age gradients of net trade credit lending hold separately within each size quartile (panel a); similarly, the size gradients hold separately within each age quartile (panel b). In addition, within each age or size group, firms relying more on external financing have a lower net AR/sales level and are less likely to be net lenders of trade credit.[3] Further, the age gradients are less steep among large firms than small firms; and the size gradients are less steep among old than young firms. For instance, within the smallest quartile of firms (size Q1), the difference in average net AR/sales between the youngest (age Q1) and oldest firms (age Q4) is -0.51. In contrast, within the largest quartile (size Q4), the difference is only -0.05, comparing the same two age groups. These patterns are consistent with the fact that firm size

[3]See appendix figure A.2 for the analogous figures for AR and AP.

(a) Within each size quartile



(b) Within each age quartile

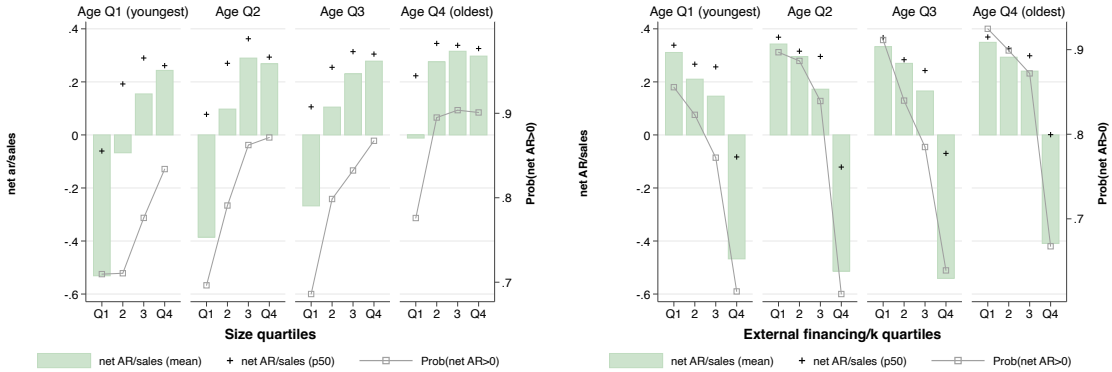


Figure 2: Net AR within each age or size quartile

Notes: The sample includes all non-financial firms in the Compustat dataset for 2000-2007. The figures plot net AR/sales (bars and +) and the probability of being a net trade credit lender (gray lines) within each quartile of age or size distribution. Panel (a) plots net AR over firm size (left) or reliance on external financing (right) within each age quartile. Panel (b) plots net AR over firm age (left) or reliance on external financing (right) within each size quartile. A similar plot for AR/sales and AP/sales separately can be found in appendix figure A.2.

and age both contain relevant information about financing frictions (Almeida and Campello, 2007). Taken together, the documented correlations conditional on firm size or age provide further support for the intuitive relationship between financial constraints and trade credit choices.

While informative, these statistics do not take into account any industry-specific trade credit practices. A firm's trade credit decisions may be affected by industry-level characteristics, such as the industry's position in the supply chain (Kim and Shin, 2012) or time-series

variations. To control for these, I estimate the following equation:

$$y_{it} = \alpha_1 \log(\text{age})_{it} + \alpha_2 \log(\text{total asset})_{it} + \alpha_3 \text{non-borrower}_{it} + \phi_s + \varphi_t + \epsilon_{it}, \tag{1}$$

where y_{it} is one of four measures of trade credit for firm i in year t: AP/sales, AR/sales, net AR/sales and a net trade credit lender indicator $I_{\text{net AR}>0}$. The independent variables include the (logged) firm age, (logged) total assets, and a dummy variable indicating whether this firm is a non-borrower (capital expenditure $\leq$ available funds). The regression includes a set of industry (ϕ_s) and quarter fixed effects (φ_t).

Table 1: Trade credit and firm characteristics

	(1)	(2)	(3)	(4)
Firm size (log size)	-0.073***	0.008***	0.077***	0.039***
	(0.005)	(0.002)	(0.005)	(0.003)
Firm age (log years)	-0.058***	0.007	0.063***	0.044***
	(0.011)	(0.005)	(0.011)	(0.007)
Non-borrower	-0.242***	-0.031***	0.200***	0.115***
	(0.013)	(0.006)	(0.013)	(0.010)
Dependent variable	AP/Sales	AR/Sales	NetAR/Sales	$\mathbf{I}_{\text{NetAR}>0}$
Industry, quarter FEs	Y	Y	Y	Y
N	29430	29285	29338	29338
R2	0.163	0.093	0.195	0.183

Notes: The table displays results from regression equation 1. The sample includes all non-financial firms in the Compustat dataset for the period 2000-2007. All regressions include a set of 2-digit sic industry and quarter fixed effects. Standard errors are clustered at the firm level and shown in parentheses.

The regression results in table 1 corroborate the findings in the previous figures. After controlling for industry and time fixed effects, net trade credit lending (net AR/sales) is significantly higher among older, larger and non-borrower firms. The estimated coefficients are 0.077 and 0.063 on firm size and age, while the coefficient on non-borrower (firms not relying on external financing) is 0.2, with all three highly significant at the 1% level (column 3). A very similar pattern emerges in column (4), where the dependent variable is an indicator of whether the firm is a net trade credit lender (net AR>0). Furthermore, consistent with the findings in the previous figures, net trade credit lending is again driven mostly by borrowing

trade credit (i.e., AP) rather than lending (i.e., AR). The estimates in column (1) show that firm size, age, and being a non-borrower have a negative and significant correlation with the size of AP/sales. On the other hand, the estimated coefficients for AR/sales are insignificant for firm age, and their magnitude is only approximately 1/10 of the coefficients for AP/sales (column 2).[4]

The facts documented in this section are consistent with previous findings in the literature emphasizing the financial motives behind trade credit (Meltzer, 1960, Schwartz, 1974, and Petersen and Rajan, 1997). Compared with the existing evidence, AR seems to vary little with firm size and age shown in figure 1. In comparison, for instance, Petersen and Rajan (1997) found a positive relationship between AR and firm age/size using the Surveys of Small Business Finances.[5] Despite this difference, there exists strong collective evidence, documented here and in previous papers, of trade credit flowing from unconstrained to constrained firms in net terms.

2.2 Trade credit and financial shocks

Given the financial motive for trade credit documented above, one would expect that firms would borrow more trade credit from their suppliers and/or reduce their lending of trade credit when facing a negative financial shock.

I provide a simple test of this hypothesis by examining the events around the Lehman Brothers bankruptcy in 2008, using a strategy similar to that of Chodorow-Reich (2014). After the Lehman bankruptcy in 2008Q3, lending activities in the financial market contracted significantly. In particular, banks that co-syndicated more loans with Lehman Brothers also reduced their lending more than other banks (Ivashina and Scharfstein, 2010). Moreover,

[4]These results are robust to the additional controls of inventory/sales and ROA (see appendix table A.1).

[5]One possible explanation for the weaker correlation between AR and firms size/age in the Compustat dataset is public firms' good access to accounts receivable financing, which reduces the liquidity loss from trade credit lending. According to the DealScan dataset, in the syndicated market where many public firms borrow, 46% of the secured loans use AR as collateral. AR also has an average advance rate of 87%, indicating that lending one dollar of trade credit results in only a 13 cents loss of liquidity. In comparison, the advance rates are 59% for inventory and 29% for property, plant, and equipment among the DealScan loans.

because it is costly/takes time for firms to switch to a new lender (Sufi, 2007 and Chodorow-Reich, 2014), their access to financing is disrupted when the bank they usually deal with (i.e., their relationship bank) tightens their lending. In other words, firms experienced differential exposure to the Lehman shock, depending on their banks' connections to Lehman before its bankruptcy. I exploit this as an exogenous financial shock and investigate how it impacts firms' lending and borrowing of trade credit.

Data. The analysis combines the Compustat dataset and the Thomson Reuters DealScan database, which reports information about loans issued in the syndicated loan market. The syndicated market is one of the most important places for large U.S. firms to obtain funding. According to Chodorow-Reich (2014), it accounts for almost half of all commercial and industrial lending. However, during 2007–09 financial crisis, there was a severe contraction in lending activities in the syndicated market with significant declines in the number, size, and maturity of new credit facilities (including AR-collateralized facilities), as shown in figure A.3 in the appendix.

I follow Sufi (2007) and Chodorow-Reich (2014) to construct a sample of Compustat firms that borrow from the syndicated loan market and to identify firms' relationship banks before the Lehman bankruptcy. To do so, I focus on the loan facilities open from January 1, 2004, to December 31, 2006.[6] Focusing on loan facilities with a single lead lender, I use the link table provided by Chava and Roberts (2008) to match each loan facility's lead lender in the DealScan database with the borrower from the Compustat database. If a firm had only one open facility during this period, I would define the lead lender of this facility as the firm's pre-crisis relationship bank. If a firm had multiple facilities during that period, I would then define the newest facility's lead lender as its relationship bank.

Following Ivashina and Scharfstein (2010) and Chodorow-Reich (2014), I measure a firm's exposure to the Lehman shock using the fraction of its relationship bank's syndication portfo-

[6]Focusing on this three-year window allows us to identify the relationship between firms and their lenders prior to the 2007-09 financial crisis. The results are robust if I extend the period to 2008Q2, the quarter before the bankruptcy of the Lehman Brothers.

lio in which Lehman Brothers had a lead role. As shown by Ivashina and Scharfstein (2010), this measure is negatively correlated with new lending activities. The data on banks' syndication portfolios are taken from Chodorow-Reich (2014), covering the 43 most active lenders. As a result, my analysis focuses on firms whose relationship bank is one of the 43 lenders.

The above process yields a panel of 605 firm-bank pairs from 2007Q1 to 2010Q4 at the quarterly frequency. The sample is a good representation of the universe of Compustat firms in terms of sectoral composition; however, the average DealScan-Compustat firm is eight times larger in total assets than the universe of Compustat firms. As such, the DealScan-Compustat sample consists of the very largest U.S. firms with the best access to financing.

Results. Dividing firms into four quartiles based on exposure to Lehman, I find that they have similar levels of AR/sales and AP/sales over the period of 2007Q1-08Q2 (panel a of figure 3). I interpret this as evidence that, before the Lehman bankruptcy, firms' financial conditions are not systematically correlated with their banks' exposure to Lehman. In other words, the evidence suggests that the exposure to Lehman's bankruptcy can be viewed as a random event for the borrowing firms, consistent with findings in Chodorow-Reich (2014).

The overall trade credit activities contracted after the Lehman shock in the sample. Panel (b) plots the percent changes in AR/sales and AP/sales from 2007Q1-08Q2 to 2008Q3-10Q4. Average AR/sales declined for all but the least-exposed group of firms, while average AP/sales declined for all four groups.[7] Keeping in mind the overall contraction in trade credit activities, in the following exercises, I focus on examining the difference across the four groups of firms.

Results from panel (b) of figure 3 confirm the hypothesis. The changes in AR and AP are correlated with firms' exposure to the Lehman shock in an intuitive way. Compared with firms with less exposure to Lehman, those with a higher exposure (i) cut trade credit

[7] Love et al. (2007) documented a pronounced decline in trade credit that lasted for several years during the Asian financial crisis. Similarly, I find the decrease in trade credit activities lasted for two years after the Lehman bankruptcy, which motivates the choice of the window for comparison. The pattern in panel (b) holds if I look at a shorter window after the Lehman shock.

lending (AR/sales) by more, and (ii) reduce trade credit borrowing (AP/sales) by less. For instance, average trade credit lending (AR/sales) declined by 6% among the most exposed firms, whereas it even increased by 1% among the least exposed firms. On the other hand, average trade credit borrowing (AP/sales) declined by 10% among the least exposed firms, significantly higher than the 1% decline among the most exposed ones.

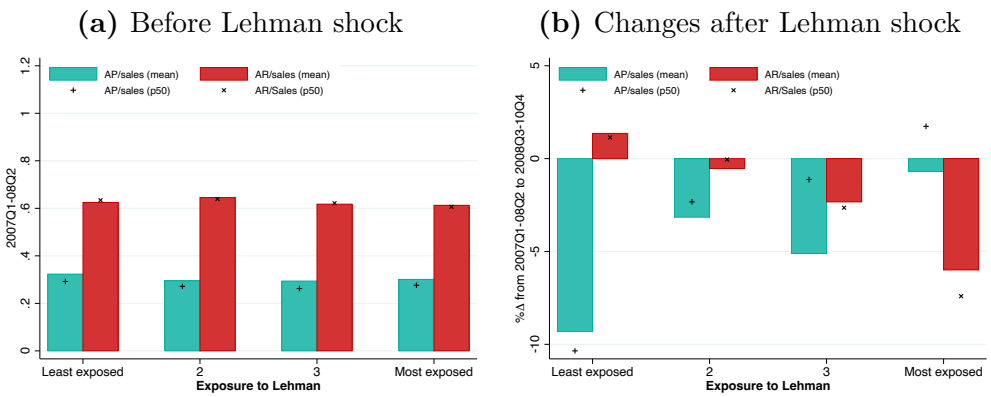


Figure 3: Trade credit before and after Lehman shock, by exposure to Lehman

Notes: This table presents the percent change in AR/sales and AP/sales from 2007Q1-2008Q3 to 2008Q4-2010Q4 by firms' exposure to Lehman through their relationship banks. The bars represent the mean and the (+ and x) represent the median AP/sales and AR/sales for each quartile.

If we use the least exposed firms as the reference group, panel (b) shows that firms facing more severe shock (high exposure to Lehman) reduce their trade credit lending while increasing their trade credit borrowing. Applying this pattern to firms in a close economy, where aggregate AR is equal to aggregate AP, one can infer that trade credit would flow, in net terms, from firms facing less severe shocks to those facing more severe shocks.

Furthermore, table 2 shows that the same pattern holds at the 25th, 50th, and 75th percentile in the AR/sales and AP/sales distribution. There were no systematic differences between the four groups of firms at these percentiles before the Lehman bankruptcy. After the bankruptcy, however, at each percentile, compared with firms with low exposure, firms with higher pre-crisis exposure to Lehman increased their borrowing of trade credit while cutting back their trade credit lending. By examining the movement at different points along the distribution, I conclude that this pattern holds consistently and is unlikely to be driven by outliers.

Table 2: Trade credit before and after Lehman shock, more moments

Panel (a): AP/Sales	2007Q1-08Q2				% Δ from 2007Q1-08Q3 to 2008Q3-10Q4			
Exposure to Lehman	mean	p25	p50	p75	mean	p25	p50	p75
Q1 (least exposed)	0.32	0.20	0.29	0.43	-9.31	-16.82	-10.35	-5.95
2	0.29	0.21	0.27	0.36	-3.05	-12.62	-2.19	-0.31
3	0.29	0.18	0.26	0.38	-5.11	-1.88	-1.12	-7.84
Q4	0.30	0.18	0.28	0.40	-0.70	0.68	1.73	1.91
Total	0.30	0.19	0.27	0.39	-4.59	-7.17	-3.39	-4.12

Panel (b): AR/Sales	2007Q1-08Q2				% Δ from 2007Q1-08Q3 to 2008Q3-10Q4			
Exposure to Lehman	mean	p25	p50	p75	mean	p25	p50	p75
Q1 (least exposed)	0.62	0.50	0.63	0.75	1.36	2.45	1.14	0.84
2	0.65	0.53	0.64	0.75	-0.53	-2.66	0.18	2.26
3	0.62	0.49	0.62	0.74	-2.33	-4.56	-2.64	-2.11
Q4	0.61	0.50	0.61	0.71	-5.99	-6.88	-7.41	-4.80
Total	0.63	0.51	0.62	0.74	-1.90	-3.65	-2.38	-0.99

Notes: This table presents the percent change in AR/sales and AP/sales from 2007Q1-2008Q3 to 2008Q4-2010Q4 by firms' exposure to Lehman through their relationship banks and along the different points in the distribution. Specifically, for each group of entrepreneurs, I compute the mean, p25, p50, and p75 values for AR/sales and AP/sales for 2007Q1-08Q2 and 2008Q3-10Q4. The table presents the percent changes in these values between these two periods. The first quartile (Q1) are firms with the lowest pre-bankruptcy exposure to Lehman.

To summarize, the evidence highlighted in this section further supports the existence of financial motives behind trade credit choices. In addition, it illustrates the transmission of shocks from the financial sector to firms through endogenous trade credit channels. First, firms pass on the negative financial shock to their customers by cutting back their trade credit lending (AR), consistent with existing findings in Jacobson and von Schedvin (2015) and Costello (2020). Second, firms can adjust the other aspect of trade credit by borrowing more from suppliers (AP), potentially mitigating the effect of negative bank credit shocks. Both margins of adjustment play a role in shaping the impact of financial shocks on the real economy.

3 Model

Motivated by the empirical evidence, this section presents a dynamic general equilibrium model with heterogeneous entrepreneurs and financial frictions. In addition to production, entrepreneurs engage in trade credit lending and borrowing. Trade credit and bank credit coexist as two sources of funding, which is a crucial departure from the standard model with financial frictions and heterogeneity.

3.1 Economic environment

Time is discrete with an infinite horizon. There is one type of good, which is sold in a perfectly competitive market and used for consumption, investment, and intermediate input. There exists a unit measure of heterogeneous entrepreneurs who use capital (k), labor (l), and intermediate inputs (x) to produce. The entrepreneurs differ by wealth (a, endogenous) and productivity (z, exogenous).

There is a measure N of homogeneous workers, who provide labor, receive wages, and consume. They have no access to asset markets and therefore consume their labor income every period; i.e., they are "hand-to-mouth."

Preferences and endowments. Worker preferences are time separable with instantaneous utility function $u(c_t^h, h_t)$ of the GHH form (Greenwood et al., 1988):

$$U^h(c^h, h) = \sum_t \beta^t u(c_t^h, h_t) = \sum_t \beta^t \left(c_t^h - \psi \frac{h_t^{1+\theta}}{1+\theta} \right)$$

where β is the discount factor, c_t^h is consumption, and h_t is labor provided by the worker.

Entrepreneur preferences are time separable with instantaneous utility function of $\log(c_t)$.

The expected utility of the entrepreneur can be written:

$$U^e(c) = \mathbb{E} \sum_t \beta^t \log(c_t),$$

where the expectation is taken over the stochastic processes of productivity z and wealth a.

Production technology. Entrepreneurs operate a decreasing returns to scale production technology ($\mu < 1$) that transforms capital, labor, and intermediate inputs into the final good:

$$y = Az \left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi \right)^\mu$$

where A is aggregate TFP and z is idiosyncratic productivity that follows an exogenous Markov process $\Gamma_z(z'|z)$.[8] Since the production function is decreasing returns to scale, there is an optimal production scale for any given productivity z.

3.2 Financing production

Entrepreneurs deposit their wealth a with perfectly competitive financial intermediaries (banks) and rent capital from them. The deposit rate is r, and the zero-profit condition gives a capital rental rate of $R = r + \delta$. In this case, the entrepreneur's net interest payment is $r(k - a)$. This structure of the capital rental market is the same as in Buera and Moll (2015) and Zetlin-Jones and Shourideh (2017). This model setup is equivalent to one where entrepreneurs own capital k and borrow an inter-temporal loan d at interest rate r to finance investment. In this case, the entrepreneur's net worth is $a = k - d$, and their interest payment is $rd = r(k - a)$, the same as the current setup.

[8] The economy admits an aggregate value-added production function $Y = \bar{A} K^{\frac{\alpha(1-\chi)\mu}{1-\chi\mu}} L^{\frac{(1-\alpha)(1-\chi)\mu}{1-\chi\mu}}$, where K and L are the aggregate capital stock and labor inputs. In the absence of financial frictions, $\bar{A}$ is a function only of A and the exogenous stationary distribution of z.

Timing At the beginning of each period, entrepreneurs enter with wealth a, and their productivity z is realized. Consistent with the timing of the model in Jermann and Quadrini (2012), I assume that, after learning their productivity draw, firms choose their inputs (k, l, x), trade credit (to be introduced below), consumption c, and savings into the next period $a' - a$.

However, as in Jermann and Quadrini (2012) entrepreneurs need to pay for the cost of their labor, capital rental, intermediate goods, consumption, and investment before receiving this period's revenue. Therefore entrepreneurs must raise an intra-temporal loan m to cover the timing mismatch in cash flows. The intra-temporal loan is repaid within the period, and there is no interest. After obtaining financing, entrepreneurs produce, purchase goods for consumption, and save for the next period.

This brings us to the end of the period. At this point, entrepreneurs decide whether to repay their loans. Crucial to this model, the two sources of financing (bank loans and trade credit) differ in their exposure to moral hazard, which gives suppliers a comparative advantage in lending inputs to their customers. More specifically, I assume that entrepreneurs can default on bank loans, but trade credit is perfectly enforceable.

When deciding whether to default, the entrepreneur's assets consist of savings a' and AR owed from customers, and its liabilities consist of bank loans m and AP owed to suppliers. Under the assumption, the AP must be repaid, but the bank loan m can be defaulted on. If the entrepreneur defaults, the bank can seize and liquidate the entrepreneur's assets (a' and AR), but the liquidation is only successful with some probability. Therefore, when contracting the loans, the bank will foresee this and limit the size of the loan based on the anticipated proceeds from liquidation:

$$m \leq \gamma_1 a' + \gamma_2 \text{AR}. \tag{2}$$

where γ_1 and γ_2 are the probability of liquidating a' and AR, respectively. Figure 4 illustrates the timing of events.

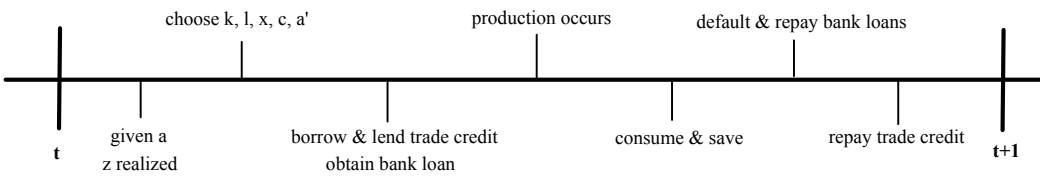


Figure 4: Timing

Trade credit. By nature, trade credit arises only in connection with the purchase of intermediate inputs. Recalling the intermediate inputs market is perfectly competitive; entrepreneurs who wish to borrow will therefore purchase inputs from the supplier who offers the most attractive trade credit terms. As a result, the trade credit interest rate will be equalized across all borrowers in equilibrium. A similar feature of the competitive input market can be found in the model of Burkart and Ellingsen (2004), which argues that the assumption is consistent with the empirical finding that trade credit terms look similar across borrowers within the same industry. As noted by Burkart and Ellingsen (2004), the trade credit interest rate reflects the financial condition of the *average* entrepreneur in the market rather than a specific seller and borrower.[9]

To formally model trade credit and input transactions, I assume a competitive market for intermediate inputs and trade credit. Entrepreneurs are price takers. They supply their output $y = Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu}$ and offer a trade credit loan of size $\text{AR} \in [0, y]$. They also purchase intermediate goods of value x and borrow trade credit of size $\text{AP} \in [0, x]$. Note that AR and AP are both non-negative and cannot exceed the value of goods sold or bought. Indeed, this tight connection between trade credit and the value of goods is an important feature that distinguishes trade credit from other types of inter-firm lending. At the end of the period, entrepreneurs collect a payment of $y - x + r^{tc}(\text{AR} - \text{AP})$ from the market, where $y - x$ is the profit from production, and $r^{tc}(\text{AR} - \text{AP})$ is the net interest from lending and borrowing trade credit. The entrepreneur's budget constraint can be written as:

$$c + a' = (1 + r)a + Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} - (r + \delta)k - wl - x + r^{tc}(\text{AR} - \text{AP}). \tag{3}$$

[9]In equilibrium, trade credit interest rates can be positive, whereas the intra-temporal bank loan has an interest rate of zero. This is because, unlike banks, some trade credit lenders (suppliers) are financially constrained, so to be willing to lend in equilibrium, they need to be compensated by a positive interest rate.

Before deriving the working capital constraint, a few remarks are in order on the modeling of trade credit. First, since the primary goal in this model is to capture the impact of trade credit on the entrepreneur's liquidity position, I abstract away from a real-world feature of trade credit. In reality, trade credit borrowing takes the form of a delay in payment rather than a direct loan. In Appendix section B.1, I construct an alternative model in which trade credit takes this form. Specifically, I assume that entrepreneurs carry their output into the following period and then decide whether to sell their goods on the spot or provide trade credit and delay receiving payment. I show this alternative specification is similar to the baseline model in capturing trade credit's impact on entrepreneurs' liquidity position. However, in this alternative model, the output of the previous period becomes an additional state variable for entrepreneurs, which makes the model computationally intractable. Therefore I opt for the current model set up to maintain tractability.

Second, the model builds on insights from the existing literature, which argues that trade credit exists because suppliers have a certain comparative advantage in lending to their customers over financial intermediaries (Biais and Gollier, 1997, Fabbri and Menichini, 2010, Burkart and Ellingsen, 2004 and Cuñat, 2007). I follow the papers emphasizing suppliers' comparative advantage in enforcing the repayment of trade credit loans. For instance, Cuñat (2007) argues that suppliers have a comparative advantage because they can stop supplying intermediate inputs.

Third, I abstract from modeling trade credit default, partly due to the difficulty of quantitatively separating default from liquidity loss in the model. In practice, firms often employ specific policies (non-recourse factoring or purchasing credit risk insurance) to manage trade credit default risk (Mian and Smith, 1992). With the help of these policies, firms can transform default risk into liquidity loss with certainty. Further, trade credit default in the data often refers to a delay of payment later than the pre-agreed dates. In sporadic cases (2% among all defaults among French firms, calculated using data from Boissay and Gropp, 2007), trade credit default means non-payment because the customer becomes insolvent. A delay in payment does not hurt an unconstrained entrepreneur by much; only the constrained ones suffer from liquidity loss – this mechanism is already incorporated in the model. As

shown later, the model is calibrated to match the quarterly data, longer than the usual trade credit terms. Thus, the calibrated model will likely have captured some of the default activities. Haven said these, existing literature has long recognized the importance of trade credit default in propagating financial shocks (see Kiyotaki and Moore, 1997, Jacobson and von Schedvin, 2015 and Mateos-Planas and Seccia, 2022). As a caveat to the analysis, I abstract from modeling default to maintain the model's tractability.

Working capital constraint. According to the timing assumption, the entrepreneur's outlays must be financed in advance by m, such that

$$m = a' - a + c + r(k - a) + \delta k + wl + x + \text{AR} - \text{AP}.$$

The loan must cover the net interest payment $r(k-a)$, labor inputs wl, intermediate inputs x, capital depreciation δk, consumption c, and plus or minus any change in assets held $a'-a$. Compared to Jermann and Quadrini (2012), however, there is one additional term $\text{AR}-\text{AP}$, which is the entrepreneur's net trade credit position.

Using the budget constraint (equation 3), I can rewrite the size of the loan as

$$m = Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} + (1 + r^{tc})(\text{AR} - \text{AP}).$$

Plugging this into the bank loan limit (inequality 2), I derive the working capital constraint:

$$Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} + (1 + r^{tc})(\text{AR} - \text{AP}) \leq \gamma_1 a' + \gamma_2 \text{AR}. \tag{4}$$

To see how trade credit affects the entrepreneur's liquidity position, consider what happens when I turn off trade credit by setting $\text{AR} = \text{AP} = 0$. In this case, the working capital constraint is simply $Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} \leq \gamma_1 a'$. Comparing this with constraint 4, I see that when trade credit is allowed, the entrepreneur's need for intra-temporal financing increases by $(1+r^{tc})(\text{AR}-\text{AP})$, the net amount lent or borrowed via trade credit, and the borrowing

limit increases by γ_2AR, the collateral value of AR.[10]

3.3 Recursive competitive equilibrium

Workers solve a static optimization problem, which can be written as:

$$\max_{c^h,h} c^h - \psi \frac{h^{1+\theta}}{1+\theta}, \ s.t. \ c^h = wh. \tag{5}$$

The entrepreneur solves a dynamic problem, choosing production inputs (k, l, x), trade credit (AR, AP), consumption (c), and next period wealth (a'). These choices are subject to the budget constraint (equation 7), the working capital constraint (inequality 8), and the trade credit constraints (inequalities 9 and 10). I also require the entrepreneur's wealth to be non-negative.

I can write the entrepreneur's problem recursively as follows:

$$V(a,z) = \max_{c,k,l,x,\mathrm{AR},\mathrm{AP},a'} \log(c) + \beta \mathbb{E}_{z'|z} V(a',z'), \tag{6}$$

$$\begin{aligned} s.t. \quad & c + a' = (1+r)a + Az\left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi\right)^\mu - (r+\delta)k - wl - x \\ & + r^{tc}(\mathrm{AR} - \mathrm{AP}), \end{aligned} \tag{7}$$

$$Az\left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi\right)^\mu + (1+r^{tc})(\mathrm{AR} - \mathrm{AP}) \leq \gamma_1 a' + \gamma_2 \mathrm{AR}, \tag{8}$$

$$0 \leq \mathrm{AR} \leq Az\left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi\right)^\mu, \tag{9}$$

$$0 \leq \mathrm{AP} \leq x, \tag{10}$$

$$a' \geq 0.$$

Recursive competitive equilibrium. The recursive competitive equilibrium consists of interest rate r, wage w, and trade credit rate r^{tc}; entrepreneur value function $V(a,z)$; en-

[10] The working capital constraint indicates that net trade credit interest $r^{tc}(\mathrm{AR} - \mathrm{AP})$ needs to be covered by the intra-temporal loan. This is because, according to the timing of the events, $r^{tc}(\mathrm{AR} - \mathrm{AP})$ is budgeted as part of the current-period income to cover expenditure; therefore, it needs to be financed beforehand.

trepreneur policy functions $c(a,z)$, $k(a,z)$, $l(a,z)$, $x(a,z)$, $\mathrm{AR}(a,z)$, $\mathrm{AP}(a,z)$ and $a'(a,z)$; worker consumption and hours worked (c^h, h); and a stationary distribution $\Phi(a,z)$, such that

(i) Given prices, the value function and policy functions solve entrepreneur's problem 6.

(ii) Given prices, consumption and hours worked solve the worker's problem 5.

(iii) Given prices, policy functions, and the stationary distribution, all markets clear

$$\text{[labor]} \qquad \int l(a,z)d\Phi(a,z) = Nh,$$

$$\text{[capital rental]} \qquad \int k(a,z)d\Phi(a,z) = \int a d\Phi(a,z),$$

$$\text{[trade credit]} \qquad \int \mathrm{AR}(a,z)d\Phi(a,z) = \int \mathrm{AP}(a,z)d\Phi(a,z),$$

$$\text{[goods]} \qquad \int y(a,z)d\Phi(a,z) = Nc^h + \int [c(a,z) + \delta k(a,z) + x(a,z)]d\Phi(a,z).$$

(iv) The evolution of distribution across entrepreneurs satisfies:

$$\phi(a',z') = \int \mathbb{I}_{a'=a'(a,z)}\Gamma_z(z'|z)d\Phi(a,z).$$

3.4 Optimal trade credit choices

Let $F(k,l,x) = ((k^\alpha l^{1-\alpha})^{1-\chi}x^\chi)^\mu$ denote the production function. The Lagrangian of the entrepreneur's problem can be written as

$$\begin{aligned}
\mathcal{L} = &\log((1+r)a + AzF(k,l,x) - (r+\delta)k - wl - x + r^{tc}(\mathrm{AR}-\mathrm{AP}) - a') \\
&+ \beta\mathbb{E}_{z'|z}V(a',z') + \xi(\gamma_1 a' + \gamma_2\mathrm{AR} - AzF(k,l,x) - (1+r^{tc})(\mathrm{AR}-\mathrm{AP})) \\
&+ \chi_1(AzF(k,l,x) - \mathrm{AR}) + \chi_2\mathrm{AR} \\
&+ \chi_3(x - \mathrm{AP}) + \chi_4\mathrm{AP} \\
&+ \tau a'.
\end{aligned}$$

where ξ, $\chi_1, \chi_2, \chi_3, \chi_4$ and τ are the corresponding Lagrange multipliers for the working capital constraint, the trade credit constraints, and the non-negative wealth constraint, respectively.

The FOCs of this optimization problem can be found in appendix section B.2. For exposition purposes, I reproduce the FOCs w.r.t. AR and AP here:

$$\text{[AR]} \qquad \frac{1}{c} r^{tc} = \xi(1 + r^{tc} - \gamma_2) + \chi_1 - \chi_2, \tag{11}$$

$$\text{[AP]} \qquad \frac{1}{c} r^{tc} = \xi(1 + r^{tc}) - \chi_3 + \chi_4. \tag{12}$$

With the help of the FOCs, I will show in the following proposition that the optimal choice of trade credit follows a simple cut-off rule in entrepreneurs' wealth.

Proposition 1. *For given productivity z, there exist three cut-off values for entrepreneur wealth: $a_b(z)$, $a_{AR}(z)$, and $a_{AP}(z)$, such that*

1. *The working capital constraint is binding if and only if $a \leq a_b(z)$.*
2. *$AR > 0$ if and only if $a > a_{AR}(z)$.*
3. *$AP > 0$ if and only if $a < a_{AP}(z)$.*

Proof. See Appendix B.3.1. □

The first wealth threshold $a_b(z)$ separates constrained from unconstrained entrepreneurs. Specifically, given productivity z, entrepreneurs become unconstrained when their wealth level exceeds threshold $a_b(z)$. One can show the Lagrange multiplier on working capital, ξ, which represents the shadow value of liquidity, declines monotonically with wealth for a given z, with ξ reaching zero when wealth is high enough, and the entrepreneur becomes unconstrained.[11]

[11] The decrease of ξ in wealth is not strictly monotone, and the function ξ can be divided into several segments, each associated with different value ranges for AR and AP. I discuss this in details in appendix figure C.3.

The second wealth threshold $a_{\mathrm{AR}}(z)$ separates entrepreneurs who lend trade credit from those who do not. As shown in the FOC of AR (equation 11), the marginal benefit of lending trade credit is the trade credit interest rate multiplied by the marginal utility from consumption, while the marginal cost is the liquidity loss $1 + r^{tc} - \gamma_2$ multiplied by the shadow value of ξ. Since ξ declines with wealth, there exists a threshold value for wealth above which the marginal benefit of lending trade credit exceeds the marginal cost, which corresponds to $a_{\mathrm{AR}}(z)$.

The third wealth threshold $a_{\mathrm{AP}}(z)$ separates entrepreneurs who borrow trade credit from those who do not. Using the FOC of AP (equation 12), a similar argument can be applied to understand the trade-offs behind $a_{\mathrm{AP}}(z)$.

So far, I have discussed the cut-off rules behind the three decision functions separately. Next, I will show the relationship between these three wealth cut-offs. More specifically, the following proposition shows that, under relatively intuitive conditions, these three thresholds can be ranked based on their values.

Proposition 2. *If $r^{tc} \geq 0$ and $\gamma_2 \in (0, 1]$, the following inequality holds for any given z:*

$$a_{AR}(z) \leq a_{AP}(z) \leq a_b(z).$$

Proof. See Appendix B.3.2. □

Figure 5 plots a possible shape of the cut-off functions described in the propositions. The figure shows that the state space (a, z) is divided into regions with different trade credit choices and financial constraints. I make the following observations about entrepreneurs and their trade credit choices: (i) all unconstrained entrepreneurs provide AR to their customers, (ii) some entrepreneurs give AR to their customers even if they are constrained, (iii) unconstrained entrepreneurs do not take AP from their suppliers, and (iv) some entrepreneurs may give and receive trade credit simultaneously. The critical condition for (iv) to hold is that $\gamma_2 > 0$, i.e., AR can be used as collateral to borrow bank loans.

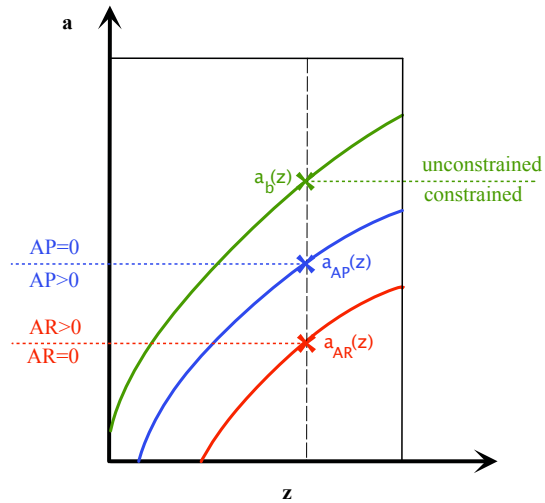


Figure 5: The cut-off property of trade credit choices

Notes: This figure shows the cut-off property of trade credit choices for entrepreneurs. The three threshold functions $a_b(z)$ (green), $a_{\text{AP}}(z)$ (blue), and $a_{\text{AR}}(z)$ (red) intersect with the vertical line at three points, which are the threshold value of wealth that separate constrained entrepreneurs from unconstrained ones, entrepreneurs who borrow trade credit and those who do not, and entrepreneurs who lend trade credit and those who do not, respectively.

4 Quantitative Analysis

This section examines the quantitative implication of the model. I begin by discussing the calibration strategy and the results. Following it, I highlight the key mechanisms of the model and show how it generates outcomes consistent with the motivating facts.

4.1 Calibration

The model is calibrated to match key features in the US data.[12] I assume that each period in the model corresponds to one quarter. Since some data moments are only available at the annual frequency, calibrating to these data moments requires a temporal aggregation of the variables. For the stock variables, such as capital and labor, I take the average of the quarterly variables to construct the annual variables; for the flow variables, such as profit and output, I take the sum of the quarterly variables to create annual variables.

[12]See appendix section C.1 for the algorithms to solve the model.

I discuss the calibration exercises in two parts: (i) parameters calibrated outside the model and (ii) parameters calibrated to target features in the data.

Parameters calibrated outside the model. I first set the values of several parameters outside the model following the standard practice of the literature. As shown in panel (a) of table 3, I set $\theta = 0.5$, which gives a Frisch elasticity of 2, a common value used in the literature and is well within the range of macro estimates (Chetty et al., 2011 and Keane and Rogerson, 2012). I fix the capital share $\alpha = 1/3$ in the production function. Consequently, the labor share is 2/3. Following Jones (2011), I fix χ so that the input's share in aggregate output is 0.43. Since the share of entrepreneurs in the data is around 10 percent and the measure of entrepreneurs in the model is 1, I set the workers' measure at $N = 9$. The capital depreciation rate δ is chosen to be 0.025, so the annual depreciation rate of capital is approximately 10 percent.

Table 3: Calibration strategy

Parameter		Value	Target/Source
Pre-determined			
θ	Frisher elasticity	0.50	standard
α	capital income share	0.33	capital share of 1/3
χ	intermediate goods share	0.43	Jones (2013)
N	measure of workers	9	share of entrepreneurs
δ	depreciation rate	0.025	10% annual depreciation rate
Calibrated			
β	discount factor	0.98	4% annual risk-free interest rate
ψ	disutility from working	0.39	hours worked
μ	scale parameter	0.85	top 10 employment share
ρ_z	persistence of z	0.92	s.d. in employment growth
σ_z	s.d. of the innovation	0.14	1-year autocorrelation of profit rate
γ_1	collateral value of wealth	0.3	ratio of debt to non-financial asset
γ_2	collateral value of AR	0.69	ratio of AR to debt

Notes: Panel (a) of this table lists all parameters that are set following the standard literature. Panel (b) of this table lists parameters that are calibrated to match specific features in the data.

Parameters targeting features in the data. I calibrate the remaining parameters to target specific data moments. The model assumes that the idiosyncratic productivity shock

follows a discrete Markov process. I set up the parameters of this process to resemble an AR(1) log-normal process: $\log z_{it} = \rho_z \log z_{it-1} + \sigma_z \epsilon_{it}$ where $\epsilon_{it} \sim \mathcal{N}(0,1)$. This way, the process is characterized by its persistence ρ_z and the s.d. of the innovation σ_z. I follow Rouwenhorst (1995) to map this AR(1) process into the discrete Markov process assumed in the model.

Seven parameters remain, and I calibrate them jointly to match seven data moments. These parameters are: (i) ψ, the disutility of providing labor in workers' utility function, (ii) β, the discounting factor of entrepreneurs, (iii) μ, the span-of-control parameter in the production function, (iv) ρ_z, the persistence of the idiosyncratic productivity process, (v) σ_z, the s.d. of the innovation term, (vi) γ_1, the collateral value of a', and (vii) γ_2, the collateral value of AR.

Although the parameters are calibrated jointly, each is intuitively linked to a particular moment. The disutility from working, ψ, is closely related to the average hours worked in the data. I calibrate ψ so that 30 percent of workers' time is spent working, i.e., $h = 0.3$. The discounting factor β is calibrated to match an annual risk-free interest rate of 0.04.

The span-of-control parameter μ is sensitive to the employment share of the largest firms. I pick μ so that the model matches the employment share of the top 10 percentile of firms. I calculate this data moment using the Business Dynamics Statistics dataset. On average, the top 10 percent of firms in the US employ 67% of all workers.

I choose ρ_z and σ_z so that our model reproduces two moments documented in the literature: (i) the s.d. of annual employment growth of 0.38 documented by Davis et al. (2007), and (ii) the 1-year auto-correlation of annual profit rate documented by Gourio (2018) using Compustat. Depending on the sample and the specification, Gourio (2018) estimates that the auto-correlation ranges from 0.68 to 0.79.

The calibrated productivity process has a persistence of 0.92 and a s.d. of innovation of 0.14. Under this calibration, the s.d. of employment growth and the 1-year auto-correlation of the profit rate in the model are 0.38 and 0.76, respectively. The annualized productivity

process has a persistence of 0.71 and a s.d. of 0.34, comparable to the estimated process in Imrohoroglu and Tuzel (2014) using Compustat data, which has a persistence of 0.70 and a s.d. of 0.375.

Table 4: Model and data moments

	Model	Data
Targeted		
Hours worked	0.30	0.30
Risk-free interest rate	0.04	0.04
Top 10 percentile employment share	0.67	0.67
1-year auto-correlation in profit	0.76	[0.68,0.79]
S.D. of annual employment growth	0.38	0.38
Ratio of debt to non-financial asset	0.36	0.36
Ratio of AR to debt	0.32	0.32
Untargeted		
Corr. between net AR/sales and log size	0.27	0.27
Corr. between Prob($\mathbb{I}_{\text{net AR}>0}$) and log size	0.27	0.24

Notes: The top 10 percentile employment share is computed using the Business Dynamics Statistics data. 1-year auto-correlation in profit rate follows the estimates in Gourio (2018). The s.d. of employment growth is estimated by Davis et al. (2007) for 2001. I take credit market liability from Flow of Funds Table L.103 line 23 and nonfinancial asset size from Flow of Funds Table B.103 line 2. Trade receivable is taken from Flow of Funds Table L.103 line 15.

I pick γ_1, the collateral constraint on wealth a', to match the ratio of credit market liabilities to non-financial assets in the US non-financial corporate sector. I follow Jermann and Quadrini (2012) to construct this data moment using the Flow of Funds dataset. Specifically, credit market liability equals the sum of "debt securities" and "loans." The non-financial assets include equipment, real estate, and intellectual property product (IPP). The ratio of credit market liabilities to non-financial assets averaged 0.36 from 2004 to 2006. At the same time, the collateral value of AR, γ_2, is calibrated to match the ratio of AR to credit market liabilities of the US non-financial corporate sector, which averaged 0.32 in the data during the same period. The calibration shows that $\gamma_1 = 0.3$ and $\gamma_2 = 0.69$ can match these two targets. Table 4 examines the implied moments of the model against the data and shows that the model matches data features well.

To further evaluate model performance, table 4 shows the correlation between net AR

to sales ratio and logged firms size (measured by capital) is 0.27 in the model, the same as that in the Compustat data. Additionally, the correlation between the probability of being a net trade credit lender and log size is 0.27 in the model, comparable to that of 0.24 in the data.[13]

4.2 Model implications related to the motivating facts

The model emphasizes the vital role of firms' heterogeneous financial conditions in determining the choice of trade credit. In this section, I argue that the model's predictions are consistent with the patterns highlighted in the empirical section and that heterogeneity in financial constraints is an important driver of the allocation of trade credit across firms.

4.2.1 Trade credit, financial constraint, and entrepreneur heterogeneity

To examine relationship between trade credit choice and firm-level financial constraints in the model, I begin by examining entrepreneurs' optimal policy functions. First, recall that the Lagrange multiplier ξ in entrepreneurs' optimization problem represents the marginal value of liquidity. Panel (a) of figure 6 shows that ξ is a decreasing in wealth for any given productivity level, and it shifts upward when productivity increases, indicating that, conditional on having the same wealth, higher-productivity entrepreneurs are more constrained than low-productivity entrepreneurs. Overall, this figure also illustrates how the model maps the (a, z) space into different values of ξ, generating heterogeneity in financial constraints.

Next, panels (b) and (c) in figure 6 plot firms' choice of trade credit – AR/y and AP/x – as a function of wealth. Both policy functions are bounded between 0 and 1.[14] Given

[13]Note that I focus on the relationship between net trade credit lending and firm size because the absolute levels of AR and AP are not entirely comparable in the data and model. In the Compustat data, the aggregate AR-to-AP ratio is 1.7, indicating that, as a whole, the public firms are net trade credit lenders to the rest of the economy. This differs markedly from the model economy, where the equilibrium condition requires aggregate AR equal to AP.

[14]The policy functions in panels (b) and (c) show that some entrepreneurs choose not to lend or borrow trade credit (AR = 0 or AP = 0), which differs from the data, where it is rare to see firms with zero AR or AP on their balance sheet. One way to reconcile this discrepancy is to notice that the empirical measure

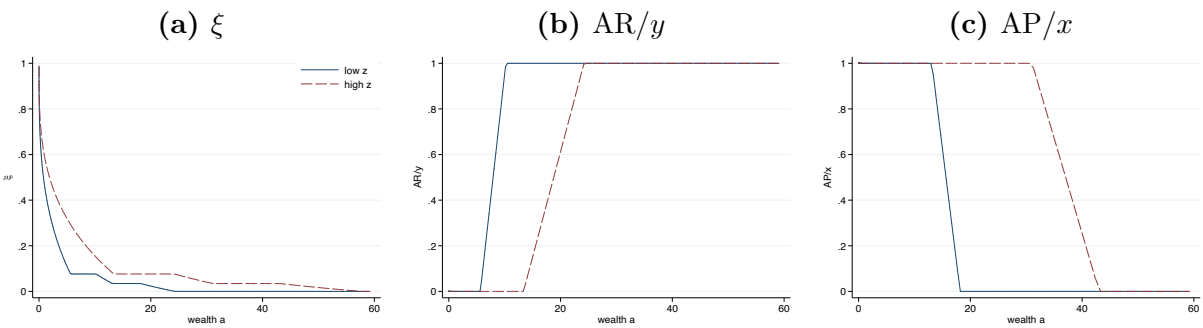


Figure 6: Policy functions for low and high values of z

Notes: Panel (a) plots the liquidity value ξ as a function of entrepreneur wealth a for a given productivity level (high or low). Panel (b) and (c) plot the lending of trade credit (i.e., AR/y) and the borrowing of trade credit (i.e., AP/x) as a function of wealth a, respectively.

productivity, trade credit lending AR/y increases with wealth, while trade credit borrowing AP/x decreases with wealth. As productivity increases, both functions shift outward. Compared with low-productivity entrepreneurs, high-productivity entrepreneurs with the same wealth, who are more financially constrained, would borrow more trade credit from suppliers and lend less to customers.

Table 5: Trade credit by size quartiles in the model

Size	ξ	P(AR>AP)	AR/sales	AP/sales
Smallest	0.134	0.553	0.549	0.214
2nd quartile	0.123	0.599	0.590	0.199
3rd quartile	0.112	0.627	0.619	0.190
Largest	0.095	0.674	0.669	0.164

Notes: This table presents the average liquidity value, net trade credit lender share, average AR/sales, and average AP/sales in each of the four size quartiles.

Having discussed the policy functions, I now turn to examine the size relationship of trade credit in the steady state distribution. Table 5 divides entrepreneurs into four size quartiles (measured by wealth) and presents each group's average trade credit choice. Results show that smaller entrepreneurs value liquidity more than larger entrepreneurs (i.e.,

of AR and AP is a snapshot of firms' activities; hence they likely reflect the averages of AR and AP over all the sales and purchases that overlapped in time. Similarly, suppose AR and AP in the model reflect the averages over several sales. In that case, I find that the policy functions no longer take corner solutions and thus can better mimic the observed patterns in the data (for more details, see appendix C.3).

their average ξ is higher) and therefore are more financially constrained. Meanwhile, larger entrepreneurs are more likely to be net trade credit lenders, which is to be expected due to the intuitive relationship between financial constraint and trade credit choices, as discussed above. Furthermore, the average AR/sales increases with size, whereas the average AP/sales decreases with size. Overall, the model generates a cross-sectional size relationship broadly consistent with observed patterns in the data.

4.2.2 Heterogeneous exposure to financial shocks

Next, I study how entrepreneurs respond when they experience different degrees of financial shocks, as firms did after the bankruptcy of Lehman Brothers. I do so by introducing an unexpected negative financial shock to the economy, modeled as a reduction to the collateral values γ_1 and γ_2. Crucial to this experiment is that the magnitude of the shock differs across four ex-ante identical groups of entrepreneurs: while group 1's collateral values remain unchanged, groups 2, 3, and 4 experience a reduction in γ_1 and γ_2 of 5%, 10%, and 15%, respectively. Furthermore, the shock is short-lived; they last for one period, and the collateral values revert to the pre-shock level afterward.[15] I solve the full transitional dynamics following the shock and present the results based on the period when the unexpected shock hits the economy.

Figure 7 displays policy functions before and after the shock for (i) group 1, the unexposed group, whose collateral value remains unchanged, and (ii) group 4, facing a 15% reduction in collateral values, i.e., the most exposed group.

Panel (b) of figure 7 shows that, after the shock, there is a leftward shift in AR/y of the unexposed group and a rightward shift of the most exposed group. In other words, not surprisingly, the unexposed entrepreneurs increase while the exposed ones cut back their lending of trade credit. Interestingly, as shown in panel (c), there is a slightly different pattern for AP/x. The curve of AP/x shifts to the right – the borrowing of trade credit increases – for both unexposed and exposed entrepreneurs. Importantly, the increase in

[15] The persistence of the shock does not affect the qualitative results of the experiment, which is our focus.

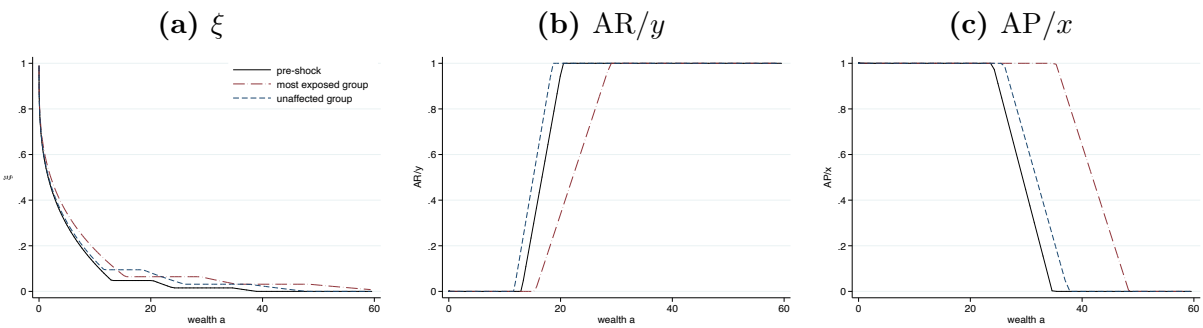


Figure 7: Shifts in policy functions by severity of the financial shock

Notes: These figures illustrate the shift in policy function after the negative shock for the unexposed group (blue line) and the most exposed group (red line). The black line shows the pre-shock policy function. Panel (a) plots the liquidity value as a function of wealth for given productivity. Panels (b) and (c) show the lending and borrowing of trade credit, respectively.

AP/x is more significant for the exposed than the unexposed entrepreneurs.

The fact that the unexposed group of entrepreneurs increases their borrowing of trade credit (panel c, blue line) reveals the intertwined nature of AR, AP, and production in equilibrium. As will become clear in the latter section, trade credit becomes more expensive following a negative financial shock due to decreased supply and increased demand for trade credit. In turn, the higher trade credit interest rate has two opposing effects on the choice of AP. On the one hand, as trade credit becomes more costly, entrepreneurs would want to borrow less of it. On the other hand, higher trade credit interest rates also raise entrepreneurs' profitability (if they lend trade credit), increasing the optimal production scale. To expand production, some entrepreneurs need to borrow more trade credit. Panel (a) of figure 7 illustrates precisely this: the ξ function shifts upward for unexposed entrepreneurs, indicating an increased value for liquidity and leading to more trade credit borrowing. Between these two opposing forces, the second one dominates in this exercise; hence, the unexposed entrepreneurs also increase their trade credit borrowing.

Table 6 presents the changes in lending and borrowing of trade credit in equilibrium, separately for the four groups of entrepreneurs. The average AR/sales increases by 2.99% for the unexposed entrepreneurs (group 1) and decreases by 2.87%, 5.73%, and 8.6% for the three exposed groups (group 2, 3, and 4, respectively). On the other hand, panel (b) shows

that the average AP/sales increases for all four groups of entrepreneurs, and the size of the increase – from 1.64% for the unexposed group up to 12.04% for the most exposed group – is positively correlated with the magnitude of the shock. These patterns also hold within each firm-size quartile, as shown in the last four columns of the table. In summary, I find that entrepreneurs facing more severe shocks reduce their lending to customers while increasing their borrowing from suppliers, in line with the evidence documented in the data.

Table 6: Percent changes in AR and AP by exposure to financial shock and firm size

Panel (a) AR/sales

Exposure to shock	full sample	size quartiles			
		smallest	2	3	largest
Unexposed	2.99	3.71	3.20	2.46	2.79
2	-2.87	-3.86	-2.70	-3.34	-1.74
3	-5.73	-7.39	-5.52	-6.21	-4.03
Most exposed	-8.60	-10.78	-8.57	-9.37	-6.05
Most exposed - unexposed (p.p.)	-11.59	-14.50	-11.77	-11.83	-8.83

Panel (b) AP/sales

Exposure to shock	full sample	size quartiles			
		smallest	2	3	largest
Unexposed	1.64	1.58	2.79	0.14	1.84
2	6.27	6.10	6.55	5.18	7.34
3	9.12	9.33	7.95	8.75	10.59
Most exposed	12.04	12.83	10.88	11.81	12.61
Most exposed - unexposed (p.p.)	10.40	11.25	8.09	11.67	10.77

Notes: This table displays the percent changes in AR/sales and AP/sales by the degree of exposure to the financial shock for the full sample (first column) and by size quartiles (last four columns). The last row in both panels shows the difference (in percentage points) between the most exposed and the unexposed group.

5 Aggregate Implications of Trade Credit

In this section, I explore the aggregate implications of trade credit. I begin with a more formal evaluation of the allocative role played by trade credit in the steady state. Then, I examine the aggregate implications of financial shocks – shocks to all or a fraction of entrepreneurs – and focus on how the endogenous changes in trade credit affect the aggregate economy.

5.1 Reallocation effect of trade credit in normal times

I consider two counterfactual experiments to evaluate the allocative effect of trade credit in the steady state.[16] In the first experiment, I shut down the trade credit channel by setting AR = AP = 0. In the second experiment, the trade credit channel is still shut down, but I replace trade credit with bank credit by raising the collateral value γ_1 from 0.3 to 0.41 so that the aggregate debt to capital stock ratio is the same as the benchmark model. While the first experiment quantifies the aggregate impact of the *existence* of trade credit, the second experiment examines how trade credit differs from bank credit in allocating resources across heterogeneous entrepreneurs.

Table 7 presents the differences between the benchmark and counterfactual economies in terms of the aggregate value-added output, hours, capital stock, and TFP. Shutting down trade credit leads to a 26.7% reduction in output, which can be decomposed into a 21.4% lower capital stock, a 23.1% lower hours worked, and a 9.7% lower aggregate TFP. Output is higher in the benchmark economy because trade credit relaxes the entrepreneurs' borrowing constraints and allows resources to be allocated more efficiently. This leads to higher aggregate productivity and, consequently, higher capital, labor, and output in the steady state.

Table 7: Difference between counterfactual and benchmark economy

Counterfactual economy	value added	capital	labor	TFP
(1) Shut down trade credit	-26.7%	-21.4%	-23.1%	-9.7%
(2) Replace trade credit with bank credit	-4.9%	-4.8%	-3.5%	-0.8%

Notes: This table displays the percent difference of the counterfactual economy relative to the benchmark economy. A negative number in the table suggests that the aggregate statistics of the counterfactual economy are lower than that of the benchmark economy. In the first counterfactual economy, I shut down trade credit by setting AR=AP=0 while keeping the other parameters fixed. In the second counterfactual economy, I increase γ_1 to 0.41 so that the aggregate debt-to-capital ratio is the same as the benchmark economy.

Replacing trade credit with bank credit also leads to lower aggregate economic activities, as shown in the second row of table 7, although compared with the previous experiment,

[16] See appendix section B.4 for the equilibrium definition of the counterfactual economy.

the magnitude of the decline is smaller. The aggregate output is 4.9% lower, with a 4.8% smaller capital stock, 3.5% lower total hours, and 0.8% lower aggregate TFP.

Table 8: Distribution across productivity groups: benchmark v.s. counterfactual

	Benchmark			Counterfactual (2)		
	low	median	high	low	median	high
Number of entrepreneurs	0.771	0.198	0.031	0.771	0.198	0.031
Output per entrepreneur	0.305	2.538	8.512	0.332	2.531	7.884
Wealth per entrepreneur	0.704	1.700	3.915	0.703	1.699	3.926

Notes: This table shows the distribution of entrepreneurs, wealth, and output across three productivity groups in the benchmark and the counterfactual economy in the second experiment where I replace trade credit with bank credit. Footnote 17 describes how I construct these three productivity groups. The first row shows the number of entrepreneurs in each group as a share of all entrepreneurs. The second row shows average wealth per entrepreneur in each productivity group with the aggregate wealth and number of entrepreneurs normalized to one. The third row shows average gross output per entrepreneur in each productivity group with the aggregate output and number of entrepreneurs normalized to one.

The lower aggregate TFP indicates that the allocative efficiency is worse when replacing trade credit with bank credit. Table 8 compares the distribution across three productivity groups (low, median, and high) between these two economies.[17] As shown in the first row, the share of entrepreneurs in each productivity group is identical across the two economies (because the same exogenous productivity process generates the distribution). In the benchmark economy, high-productivity entrepreneurs produce a larger share of aggregate output, despite having a lower wealth share than in the counterfactual economy. For instance, the average output (aggregate output and wealth normalized to one in both economies) of the most productive entrepreneurs is 8.0% higher in the benchmark economy (8.512) than in the counterfactual economy (7.884), whereas the average wealth is 0.3% lower (3.915 versus 3.926). The fact that output distribution is skewed towards more productive entrepreneurs in the benchmark model is consistent with its higher TFP. It indicates that trade credit performs better than bank credit in allocating resources to more productive entrepreneurs.

[17] In the numerical solution, I discretize the productivity space z into 30 grids. The median-productivity group consists of grids 17, 18, and 19, where grid 18 has the highest share of output among all grids. Consequently, the low-productivity group consists of grids 1 to 16, and the high-productivity group consists of grids 20 to 30. The results are robust under alternative grouping schemes.

5.2 Trade credit and financial shocks

I now investigate the role played by trade credit during a financial crisis – modeled as an unexpected shock to the collateral constraints of all entrepreneurs. To generate a financial shock of plausible magnitude, I reduce the collateral value γ_1 and γ_2 by 8% and 10%, respectively, to match the decrease in the credit of the U.S. non-financial corporate sector during the 2007–09 financial crisis. The shocks decay geometrically with a half-life of one year (four periods), consistent with the duration of the banking crisis in advanced economies.

(a) Credit market liability to capital stock ratio

(b) Trade credit to capital stock ratio

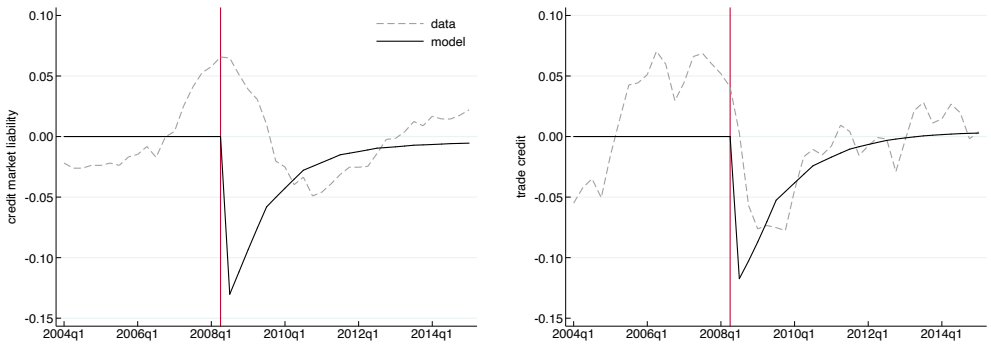


Figure 8: Dynamics of credit market liability and trade credit

Notes: The data used in the above figures are for the U.S. non-financial corporate sector. Among them, credit market liability is taken from Flow of Funds Table L.103 line 23. Trade credit is calculated as the average of trade payable (line 30 of Flow of Funds Table L.103) and trade receivable (line 15 of Flow of Funds Table L.103). The capital stock is constructed as the sum of equipment (line 46 of Flow of Funds Table B.103), intellectual property products (IPP) (line 47 of Flow of Funds Table B.103), and nonresidential structural capital (line 51 of Flow of Funds Table B.103), all valued at historical prices. Both credit market liability and trade credit to capital stock ratio are HP-filtered with a smoothing parameter of 1,600, and the percentage derivation from trend is plotted in the figures. The corresponding model moments are normalized to 0 at $t = 0$. The red vertical line corresponds to $t = 0$ in the model and 2008Q2 in the data.

Figure 8 shows the model dynamics in credit market liability and trade credit following the shock. As shown in panel (a), the model generates a decrease in the ratio of credit market liabilities to capital stock, closely matching the 11 percent decrease from peak to trough in the data. Panel (b) shows that the decline in the ratio of trade credit to capital stock is approximately 11 percent, slightly lower than the 12 percent decrease in the data. The vertical dash line marks 2008Q2, the pre-crisis peak of credit market liability to capital

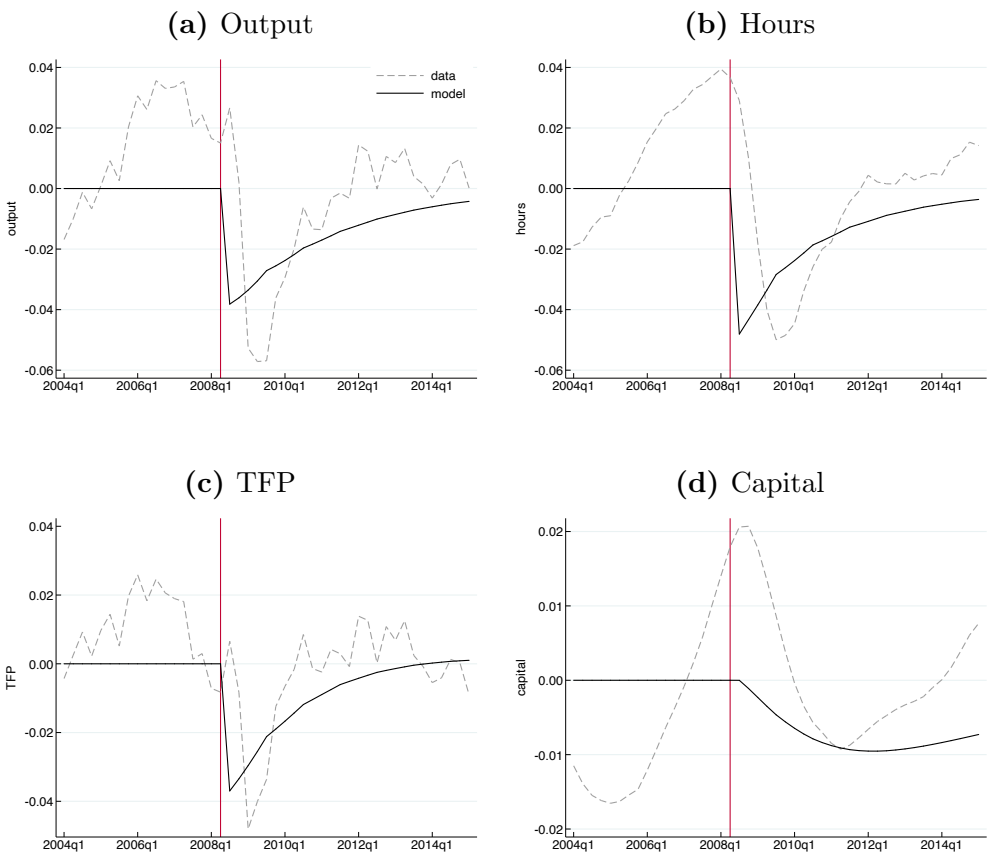


Figure 9: Dynamics of the aggregate variables

Notes: The data used in the above figures are for the U.S. nonfinancial corporate sector. Among them, output (gross value added) is taken from NIPA Table 1.14 line 17. Data for hours worked is an index taken from the Bureau of Labor Statistics Labor Productivity and Costs database (BLS code PRS88003033). Data for capital stock are constructed in the same way as Figure 8. TFP is then constructed as a Solow-type residual using output, hours, and capital stock. The corresponding model moments are normalized to 0 at $t = 0$. The red vertical line corresponds to $t = 0$ in the model and 2008Q2 in the data.

stock ratio.

Figure 9 shows that the model can generate a sizable recession following the crisis, accounting for a significant fraction of the peak-to-trough decrease in output, TFP, hours, and capital stock in the data.[18] Notably, output declined by 4%, approximately half of what we observe in the data. Hours decreased by 4.8% compared to an 8.6% decline in the data. The model also generated a 3.4% decline in TFP and a 1% decline in capital stock. Overall, the

[18]The vertical dash line marks 2008Q2. Despite the official NBER recession starting in 2007Q4, the initial drop in output was small. The most significant decrease in all four series happened in 2008Q3 or 2008Q4.

model dynamics following the shock are consistent with the findings of the existing literature. The decline in aggregate TFP results from credit tightening in the presence of producer heterogeneity (Buera and Moll, 2015). In addition, there is a significant contraction in hours worked, which is also found in other models involving working capital constraints (Jermann and Quadrini, 2012).

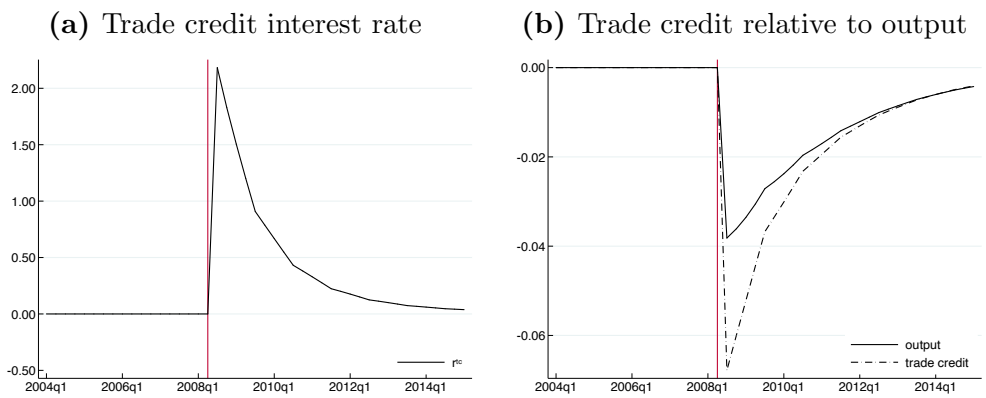


Figure 10: Trade credit interest rate and size

Notes: The two panels in this figure plot the dynamics in trade credit following the financial shock. Panel (a) shows the trade credit interest rate r^{tc} and panel (b) shows total trade credit (aggregate AR or AP) and value-added output. The values at $t = 0$ are normalized to 0. The red vertical line corresponds to $t = 0$ in the model and 2008Q2 in the data.

Figure 10 examines the dynamics of trade credit. Following the shock, entrepreneurs become more constrained: they are less willing to lend trade credit and more eager to borrow. The inward shift in the supply of trade credit and the outward shift in demand for trade credit leads to an increase in the trade credit interest rate (panel a) and, under the calibrated parameters, a decrease in trade credit relative to output (panel b). Through the lenses of the model, a direct consequence of these changes in trade credit is that, as trade credit becomes costlier and scarcer during the financial crisis, some of the constrained entrepreneurs can no longer rely on trade credit from their suppliers to finance their production. That is, the reallocation role played by trade credit during normal times, as discussed in the previous section, is impaired by the financial shock.

5.2.1 Aggregate financial shocks and the amplification effect

In this section, I quantify trade credit's role during the 2007–09 financial crisis by introducing the same financial shock to the counterfactual economy without trade credit and comparing the dynamics of the two economies following the shock. The counterfactual economy, as described in section 5.1, is recalibrated to match the steady-state debt-to-capital ratio in the benchmark model.

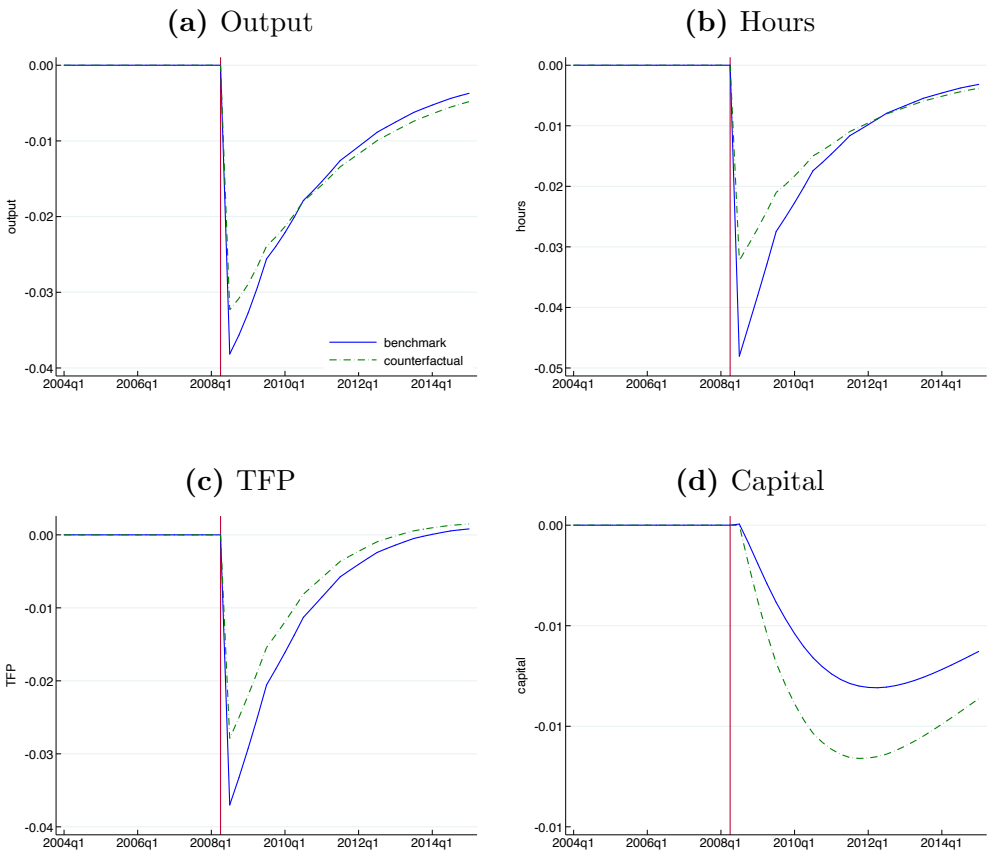


Figure 11: Dynamics of the aggregate variables: benchmark vs. counterfactual

Notes: The figures show the changes in the aggregate economy in terms of output, hours, aggregate TFP, and capital stock after the financial crisis. All lines are normalized to 0 at the beginning of the crisis. Each line in the figure represents a model economy: benchmark economy (blue) and counterfactual economy without trade credit (green).

Figure 11 shows recession generated by the same shock is greater in the benchmark economy than in the counterfactual economy, with a total output reduction 0.8 percentage

points larger, or a 17 percent larger decline than in the counterfactual.[19] In particular, the benchmark economy sees a greater decline in aggregate TFP and hours worked than the counterfactual economy. However, compared with the counterfactual case, the decline in capital stock is milder in the benchmark model. Overall, the exercise suggests that the steeper decline in TFP and hours quantitatively dominates, and consequently, the presence of trade credit amplifies the output loss during the financial crisis.

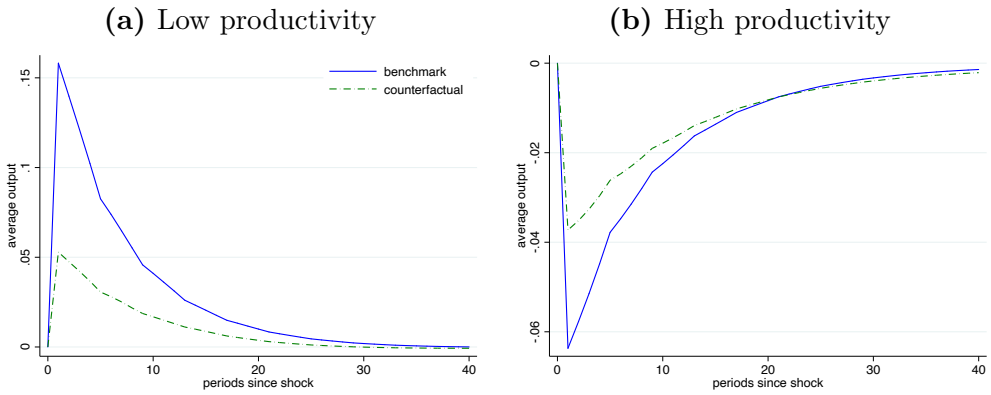


Figure 12: Average output for low and high productivity entrepreneurs

Notes: The two panels show the average output per entrepreneur of low productivity (below-median z) and high productivity (above-median z). The average output at $t = 0$ is normalized to 0. Each line in the figure represents a model economy: benchmark economy (blue) and counterfactual economy without trade credit (green).

To summarize, the benchmark and counterfactual economies differ in two important ways. First, the benchmark model features more significant productivity losses, suggesting a more severe misallocation of resources. What causes the more severe misallocation in the benchmark economy? In both economies, after the shock, resources are reallocated from high-productivity entrepreneurs to low-productivity entrepreneurs. For instance, figure 12 shows that, in both models, the average output of low-productivity entrepreneurs (those with a below-median z) increases while that of high-productivity entrepreneurs decreases. However, this reallocation toward low-productivity entrepreneurs is more pronounced in the benchmark economy, which can be attributed to two forces associated with trade credit.

[19] I also performed the quantitative analysis without recalibrating the steady state of the counterfactual model or recalibrating to match the output of the benchmark model. I found the same amplification effect of trade credit during the financial crisis, only with different magnitudes (a 15 percent and 20 percent greater decline, respectively).

On the one hand, compared with a contraction in bank credit, a contraction in trade credit is disproportionately borne by the most constrained entrepreneurs (because they are the ones using it). On the other hand, a higher trade credit interest rate benefits unconstrained entrepreneurs (lenders of trade credit) by raising their profitability. Both forces lead to more misallocation across entrepreneurs.

Second, entrepreneurs have a greater incentive to save in the benchmark model. Entrepreneurs' incentive to save depend on (i) interest rate r' and (ii) liquidity value ξ. In the benchmark model, the interest rate falls more than in the counterfactual case, as shown in panel (a) of figure 13. A lower interest rate discourages saving. On the other hand, the value of liquidity ξ increases more in the benchmark economy than in the counterfactual economy (panel b). A higher liquidity value indicates that entrepreneurs are more constrained, incentivizing them to save more to ease the borrowing constraint. Overall, the higher liquidity value's impact dominates, resulting in more savings in the benchmark economy.

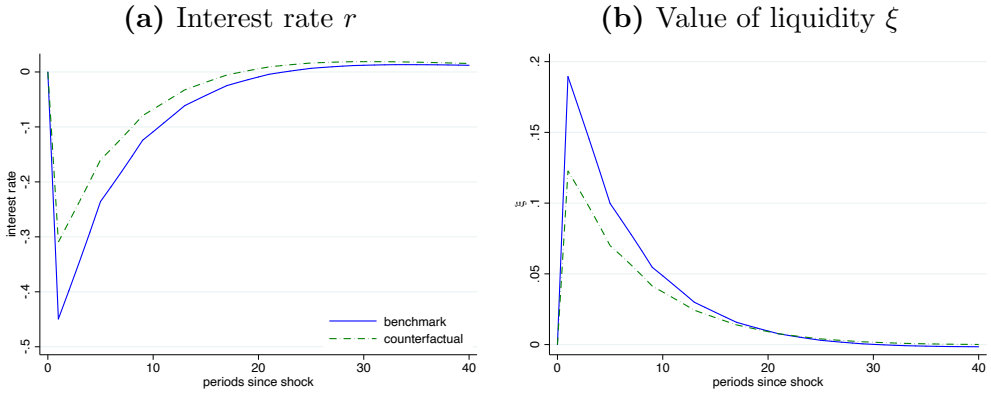


Figure 13: Incentives for saving: interest rate and value of liquidity

Notes: Panel (a) shows the dynamics of interest rate r, and panel (b) shows the value of liquidity ξ. The values at $t = 0$ is normalized to 0. Each line in the figure represents a model economy: benchmark economy (blue) and counterfactual economy without trade credit (green).

Interestingly, the counterfactual economy exhibits similar aggregate dynamics to the benchmark economy with a *fixed* trade credit interest rate (see appendix figure C.6). Setting trade credit interest rates at the pre-crisis level enables entrepreneurs to borrow trade credit at a low (pre-crisis) rate and reduces the gains from increased interest rates for trade credit lenders. In some way, it closes down the channels through which trade credit affects resource

allocation across entrepreneurs, as discussed above. Therefore, perhaps it is not surprising that the model with fixed trade credit interest rate generates similar aggregate dynamics as the counterfactual economy; hence, I interpret this as further evidence of the role played by trade credit in affecting allocation across entrepreneurs.

5.2.2 Financial shocks to a fraction of entrepreneurs and the mitigation effect

The previous section illustrates how changes in trade credit amplify an aggregate financial shock that affects *all* entrepreneurs. Suppose instead that only a fraction of entrepreneurs are affected by the financial shock; what role does trade credit play here? As evidenced by empirical evidence, when the financial shock is heterogeneous across entrepreneurs, trade credit flows to those facing more severe shocks. Consequently, one might expect trade credit to mitigate the impact of such shocks. In this section, I investigate the mitigation effect of trade credit by introducing the same collateral-constraint shock as in the previous section to *some*, but not all, entrepreneurs. Moreover, I randomly assign the shock to entrepreneurs; in other words, entrepreneurs face the same probability of receiving the shock ex-ante.

Figure 14 shows the (upon impact) output loss following the shock. When the shock hits a small fraction of the entrepreneurs, the benchmark economy experiences a smaller output loss than the counterfactual economy, consistent with the idea that the presence of trade credit mitigates the impact of idiosyncratic financial shocks. As more entrepreneurs are affected by the shock, however, the benchmark economy's output declines more steeply than the counterfactual economy. In fact, when the share of affected entrepreneurs exceeds a threshold – approximately 1%, the benchmark economy suffers a more significant output loss than the counterfactual case. Such a reversal indicates that the mitigation effect, which depends on trade credit flowing from unaffected to affected entrepreneurs, is most powerful when the shock only impacts a small percentage of entrepreneurs. As the shock spreads and fewer entrepreneurs can lend trade credit, trade credit's mitigation effect weakens. When the share of affected entrepreneurs exceeds a threshold value, the amplification effect of trade credit dominates the mitigation effect, resulting in a greater loss of output in the benchmark

economy. This is a rather intuitive result: as the shock affects more and more entrepreneurs, trade credit plays an increasingly similar role as it does during an aggregate financial shock.

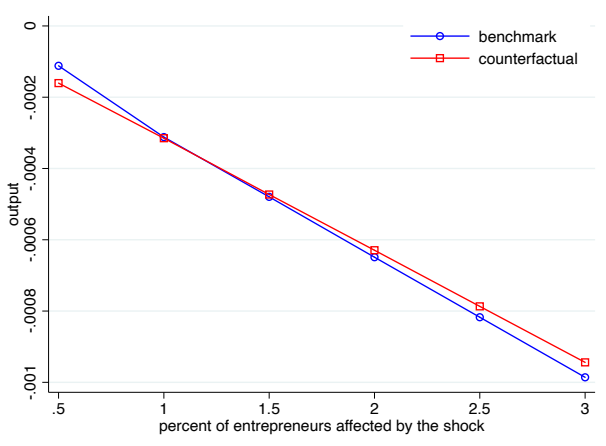


Figure 14: Output loss and share of entrepreneurs facing negative financial shocks

Notes: These figures plot the output (vertical axis) from a shock to x percent of entrepreneurs in the economy (horizontal axis). The pre-shock output is normalized to 0. The two lines in the figures represent two model economies: benchmark (blue) and counterfactual economy without trade credit (red).

Compared with the counterfactual model, the output loss *accelerates* in the benchmark model as more entrepreneurs are affected by the financial shock. In other words, in the presence of trade credit, output loss is more "backloaded" as the shock spreads gradually throughout the economy. To illustrate this, figure 15 panel (a) compares the increments in output loss in the two economies as the shock hits an increasing number of entrepreneurs. For a more transparent comparison, I normalize the output losses in both economies to 1 when the shock hits all entrepreneurs. As shown in the figure, the benchmark economy experiences an output loss of 0.182 when the first 20 percent of entrepreneurs are affected by the financial shock. Each subsequent 20 percent of entrepreneurs affected by the shock results in an increasingly larger increment in output loss, reaching 0.223 when the shock hits the last 20 percent of entrepreneurs. In contrast, the increments in output loss associated with the first 20% and last 20% of entrepreneurs are much more similar in the counterfactual model (0.194 versus 0.206).

Because the mitigation effect hinges on trade credit flowing from unaffected to affected entrepreneurs, the volume of such flows is an intuitive indicator of its strength. Panel

(b) of figures 15 plots the net trade credit borrowing (AP − AR) of the affected group of entrepreneurs. A positive value of AP − AR, as shown in the figure, indicates that these affected entrepreneurs become net trade credit borrowers after being hit by the shock. In particular, as more entrepreneurs are affected by the shock, the amount of net trade credit borrowed per affected entrepreneur decreases, reaching zero when every entrepreneur in the economy is affected. In the meantime, as the financial shock spreads, the total net trade credit borrowed by the affected entrepreneurs exhibits an inverted-U shape due to a decline in net trade credit borrowing per entrepreneur as well as an increase in the number of entrepreneurs affected. Taken together, the intuitive relationship between the volume of trade credit flows and the scope of the financial shock, as shown in the figure, further illustrates why trade credit's mitigation effect diminishes as the number of affected entrepreneurs increases.

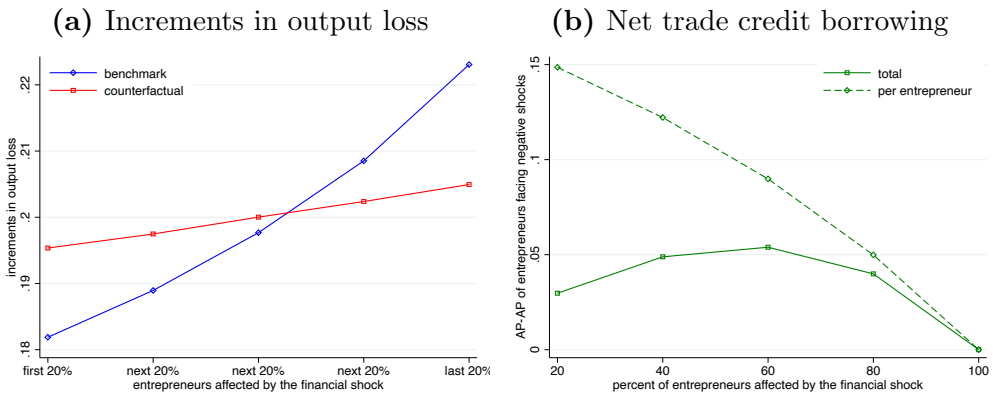


Figure 15: Acceleration of output loss and trade credit borrowing of affected entrepreneurs

Notes: Panel (a) plots the increments in output loss as the financial shock hits 20%, 40%, 60%, 80% and 100% of the entrepreneurs in the economy. The output loss when the financial shock hits 100% entrepreneurs is normalized to 1 in both economies. The two lines in the figures represent two model economies: benchmark (blue) and counterfactual economy without trade credit (red). Panel (b) plots the average and total net trade credit lending (AP − AR) against the share of entrepreneurs affected by the shock.

6 Conclusion

In this paper, I document empirical evidence supporting the existence of a financial motive behind trade credit. Motivated by this, I build trade credit into a heterogeneous entrepreneur

model with financial friction and the co-existence of trade credit and bank credit. The model generates a cross-sectional distribution of trade credit across firm sizes consistent with the empirical regularities. It also shows that trade credit flows, in net terms, to firms facing more significant financial shocks, in line with the observed pattern after the Lehman bankruptcy.

I use the model to study the aggregate implications of trade credit. Trade credit helps alleviate the misallocation of production factors. This channel, however, is dependent on suppliers' access to financing. During a financial crisis where all entrepreneurs experience a tightening in their borrowing constraint, their access to financing is disrupted, resulting in a decrease in trade credit lending, a reduction in the effectiveness of the trade credit reallocation channel, and an amplification of the original financial shock. On the other hand, when only a small fraction of entrepreneurs experience financial shocks, trade credit is shown to help entrepreneurs in distress overcome financial constraints, thus mitigating the negative impact of the shock. But this mitigation effect diminishes as financial shocks become more widespread.

Several extensions of the current framework would help us better understand trade credit and its contribution to the aggregate economy. First, several aspects of trade credit currently missing in the model are worth exploring, such as trade credit default risk and imperfect substitutability between intermediate inputs. Second, the existing empirical literature on trade credit argues that "there are multiple, not mutually exclusive, rationales for extending trade credit" (see Klapper et al., 2012). A model incorporating all these rationales – financial and non-financial – would be ideal for studying trade credit's aggregate and distributional effects. Lastly, although the model is about domestic trade credit, it can be extended into a multi-country model to study international trade finance. I leave these topics for future research.

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Online Appendix

Aggregate Fluctuations and the Role of Trade Credit

A Empirics appendix

A.1 Net accounts payable among largest firms

In this section, I examine in detail the trade credit activities of the top two size deciles firms in the Compustat dataset, motivated by the findings in Murfin and Njoroge (2015). In particular, Murfin and Njoroge (2015) finds that net payable days decrease with firm size, except for the largest two deciles of firms. In panel (a) of figure A.1, I replicate their result using our Compustat sample, where net payable days is defined as payable days (AP/cogs $\times$ 365) minus receivable days (AR/sales $\times$ 365).[20] In addition, in panel (b), I present the relationship with firm size using our preferred measure of net accounts payable (i.e., net AP/sales).

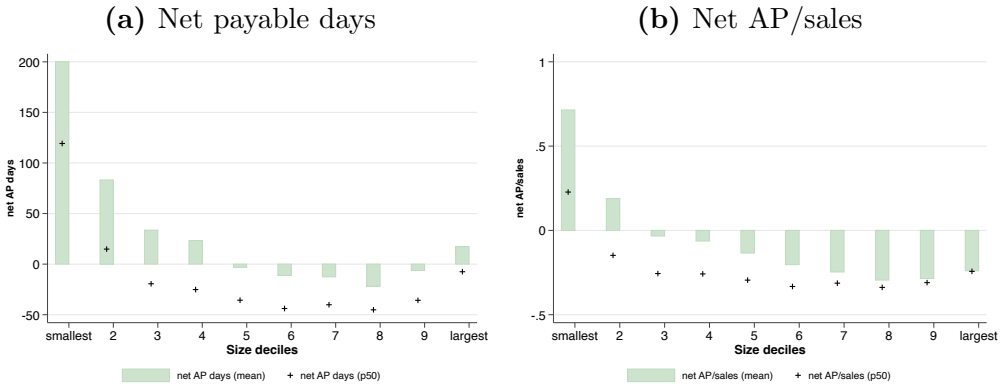


Figure A.1: Net accounts payable and firm size: two measures

Notes: The sample includes all but financial firms in the Compustat dataset for 2000-2007. The figures plot net payable days and net AP/sales within each decile of size distribution. Net payable days is defined as payable days (AP/cogs $\times$ 365) minus receivable days (AR/sales $\times$ 365). Panel (a) plots the mean and median net payable days in each size decile. Panel (b) plots each size decile's mean and median net AP/sales.

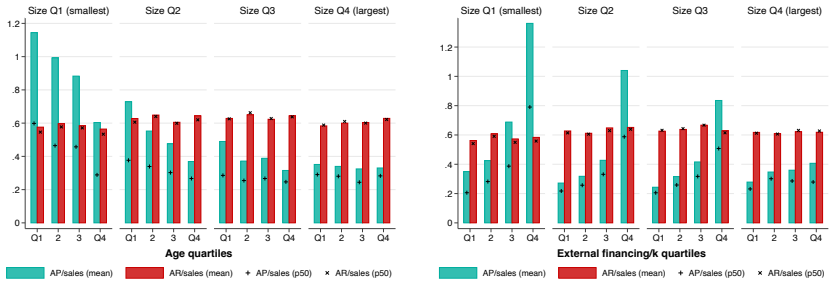
[20] Our Compustat sample excludes financial-sector firms, same as Murfin and Njoroge (2015).

The relationship between median net payables days and size deciles are very similar in panel (a) and table 1 of Murfin and Njoroge (2015). Specifically, the net payable days of the top two deciles of firms are in fact higher than some of the smaller firms. For instance, the median net payable days of the largest decile firms are even slightly higher than that of the third decile. Additionally, I show in panel (a), that this reversal, albeit not as strong, is also present for the average net payable days. The average net payable days of the largest firms, for instance, is shorter than that of the 4th decile. Similarly, there is also a slight reversal in the net AP/sales among the top two size deciles, as shown in panel (b), but the magnitude of the reversal is even less significant than net payable days. The difference between these two panels is due to the fact that in panel (a), AP is normalized by cogs (cost of goods sold) and in panel (b) it is normalized by sales.

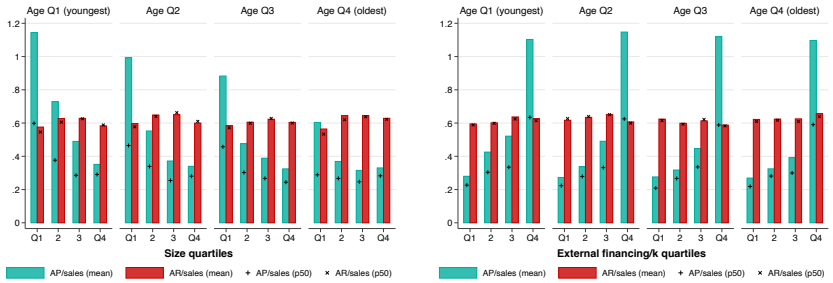
A few remarks regarding these findings are in order. First, the decline of net payable or payable in firm size is fast when firms are small, and the decline slows down as firms grow larger. In other words, the overall decline in net payable is driven by the difference between the smallest versus larger firms. This same pattern is also present in the model (see, for instance, figure C.5). Partly due to this, I note that the negative relationship between net payables and firm size is rather robust, especially comparing the smallest firms versus the rest, despite the reversal among the largest firms.

Second, as argued by Murfin and Njoroge (2015), there is a non-financial motive at play when it comes to the trade credit choices of the largest firms in Compustat. Using a sample of large retailers and their suppliers, Murfin and Njoroge (2015) show these largest buyers use a substantial amount of trade credit from their small suppliers. Similarly, Klapper et al. (2012) document that large buyers tend to get long trade credit terms from smaller suppliers. Both papers argue that these patterns are consistent with a non-financial motive to countervail frictions between suppliers and buyers related to product quality. As a caveat to our analysis, I focus on the financial motives that I view as particularly important and relevant.

A.2 Additional tables and figures



(a) Within each size quartile



(b) Within each age quartile

Figure A.2: AR and AP within each age/size quartile

Notes: The sample includes all but wholesale, retail, and financial firms in the Compustat dataset for the period 2000-2007. The figures plot AP/sales and AR/sales within each quartile of age or size distribution. Panel (a) plots AR/sales or AP/sales over firm size (left) or reliance on external financing (right) within each age quartile. Panel (b) plots AR/sales or AP/sales over firm age (left) or reliance on external financing (right) within each size quartile.

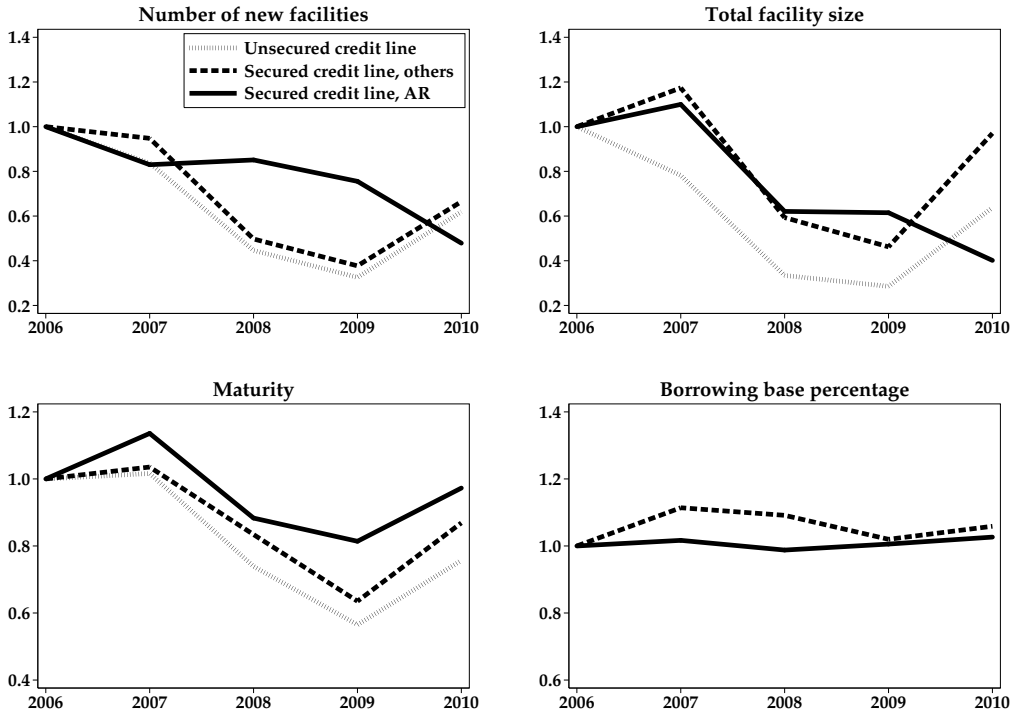


Figure A.3: Characteristics of new credit line facilities

Notes: I compute the characteristics of the newly opened credit line facilities of each year as the average of all new credit line facilities using Thomson Reuters DealScan dataset. The solid lines in these figures are credit line facilities that require accounts receivable as collateral. The dashed lines are credit line facilities that require other types of assets as collateral. The dotted lines are unsecured credit line facilities. The time series are normalized such that they are 1 in year 2006.

Table A.1: Net AR and firm characteristics, controlling for ROA & inventory

	(1)	(2)	(3)	(4)	(5)	(6)
Firm size (log size)	0.008***	0.025***	0.008***	0.021***	0.049***	0.022***
	(0.001)	(0.001)	(0.001)	(0.003)	(0.003)	(0.003)
Firm size (log years)	0.021***	0.017***	0.014***	0.058***	0.056***	0.050***
	(0.003)	(0.003)	(0.003)	(0.007)	(0.007)	(0.007)
Non-borrower	0.005	0.052***	0.008***	0.036***	0.112***	0.036***
	(0.003)	(0.003)	(0.003)	(0.008)	(0.009)	(0.008)
ROA	0.202***		0.177***	0.334***		0.320***
	(0.010)		(0.010)	(0.015)		(0.016)
Inventory/sales		0.211***	0.181***		0.362***	0.291***
		(0.033)	(0.030)		(0.067)	(0.066)
Dependent variable	NetAR/Sales	NetAR/Sales	NetAR/Sales	$\mathbf{I}_{\text{NetAR}>0}$	$\mathbf{I}_{\text{NetAR}>0}$	$\mathbf{I}_{\text{NetAR}>0}$
Industry, quarter FEs	Y	Y	Y	Y	Y	Y
N	13260	14052	12688	13260	14052	12688
R2	0.272	0.194	0.271	0.265	0.213	0.259

Notes: The table displays results from regression equation 1 while controlling for firms' ROA (returns on assets) and ratio of inventory to sales. The sample includes all non-financial firms in the Compustat dataset for the period 2000-2007. All regressions include a set of 2-digit sic industry and quarter fixed effects. Standard errors are clustered at the firm level and shown in parentheses.

B Model appendix

B.1 An alternative model of trade credit

In this section, I present an alternative model in which I treat trade credit as a delay in payment. To do so, I need to adopt a different timing in which output is carried over and sold at the beginning of the next period. The output can be sold on the spot market to generate immediate cash flow, or it can be extended as a trade credit loan.

Timing. Entrepreneurs carry over their wealth a and output y from the previous period. After the idiosyncratic productivity shock z is realized, entrepreneurs sell their output to generate cash flow: they can choose to extend some goods as trade credit loans $\text{AR} \in [0, y]$, and the remaining goods will generate an immediate cash flow $y - \text{AR}$, which can be used to finance working capital. Additionally, entrepreneurs make decisions about their current period production (k, l, x) and whether they will borrow trade credit $\text{AP} \in [0, x]$. Based on these choices, entrepreneurs obtain the required intra-temporal working capital bank loan m. At that point, entrepreneurs decide whether to default on their bank loans. If an entrepreneur decides to default, a renegotiation process occurs between the entrepreneur and the bank, with the ultimate settlement determined by the bank's expected proceeds from liquidating the entrepreneur's collateral. Using the cash flow generated at the beginning of the period and the bank loan, entrepreneurs invest in working capital and production occurs. After entrepreneurs settle the bank loan, collect AR, and repay AP, they consume c and invest in their wealth a'. This period's output is carried over into the next period as y'.

Financial frictions and the existence of trade credit Without loss of generality, assume that working capital includes interest rk, wage bills wl, and inputs x. The entrepreneurs can finance working capital using: i) bank loans, ii) cash flow generated by selling goods on the spot market, and iii) trade credit. As discussed in the benchmark model, the bank loan

limit is equal to the expected liquidation value of the collateral $\gamma_1 a + \gamma_2 \text{AR}$.[21] On the other hand, I assume that the repayment of trade credit can be enforced perfectly.

Therefore, I can write the working capital constraint of the entrepreneurs as follows:

$$rk + wl + x - \text{AP} \leq \underbrace{\gamma_1 a + \gamma_2 \text{AR}}_{bank\ loan} + \underbrace{y - \text{AR}}_{cash\ flow},$$

$$\Longrightarrow rk + wl + x + (\text{AR} - \text{AP}) - y \leq \gamma_1 a + \gamma_2 \text{AR}.$$

It captures the impact of trade credit on entrepreneurs' liquidity position similar to the benchmark model. Consider the case without trade credit, and the working capital constraint can be written as follows:

$$rk + wl + x - y \leq \gamma_1 a.$$

By comparing the two inequalities, I see that the introduction of trade credit changes the entrepreneurs' liquidity position in two ways: the entrepreneurs' needs for bank loans increase by $\text{AR} - \text{AP}$, while their bank loan limit increases by $\gamma_2 \text{AR}$, as is the case in the benchmark model. Although the two models capture a similar mechanism, compared to the benchmark model, this alternative model of trade credit is less tractable computationally as it introduces another state variable y (output carried over from the previous period).

Recursive representation of entrepreneurs' problem. To summarize, in this alternative model, entrepreneurs are characterized by three state variables (a, z, y). I write their

[21] Note that due to the difference in timing, the collateral available at the time of default is this period's wealth a and not a' as in the benchmark model.

problem recursively as follows:

$$
\begin{aligned}
V(a,z,y) = & \max u(c) + \beta \mathbb{E}_{z'|z}(a',z',y'), \\
s.t. \quad & c + a' + rk + wl + x = (1+r)a + r^{tc}(\mathrm{AR} - \mathrm{AP}) + y, \\
& rk + wl + x + (\mathrm{AR} - \mathrm{AP}) - y \leq \gamma_1 a + \gamma_2 \mathrm{AR}, \\
& 0 \leq \mathrm{AR} \leq y, \\
& 0 \leq \mathrm{AP} \leq x, \\
& y' = AzF(k,l,x), \\
& a' \geq 0.
\end{aligned}
$$

B.2 First-order conditions

Here, I present the entrepreneur's optimization problem and derive the first-order conditions (FOCs). The value function of entrepreneurs is

$$
\begin{aligned}
V(a,z) \quad = \quad & \max_{c,k,l,AR,AP,a'} \mathbb{E}_{z'|z} log(c) + \beta \mathbb{E}_{z'|z} V(a',z') \\
s.t. \quad & c + a' = (1+r)a + Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} - (r+\delta)k - wl - x \\
& +r^{tc}(AR - AP), \qquad \text{(B.1)} \\
& Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} + (1+r^{tc})(AR - AP) \leq \gamma_1 a' + \gamma_2 AR, \qquad \text{(B.2)} \\
& 0 \leq AR \leq Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu}, \\
& 0 \leq AP \leq x, \\
& a' \geq 0.
\end{aligned}
$$

Denote $F(k,l,x) = ((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi})^{\mu}$ as the production function. The Lagrangian of the

problem can be written as,

$$\begin{aligned}\mathcal{L} &= log((1+r)a + AzF(k,l,x) - (r+\delta)k - wl - x + r^{tc}(AR - AP) - a') \\ &+\beta\mathbb{E}_{z'|z}V(a',z') + \xi(\gamma_1 a' + \gamma_2 AR - AzF(k,l,x) - (1+r^{tc})(AR - AP)) \\ &+\chi_1(AzF(k,l,x) - AR) + \chi_2 AR \\ &+\chi_3(x - AP) + \chi_4 AP \\ &+\tau a'.\end{aligned} \tag{B.3}$$

The FOCs are:

$$\begin{aligned} k: &\quad AzF_k = \frac{r+\delta}{1-c\xi+c\chi_1} \\ l: &\quad AzF_l = \frac{w}{1-c\xi+c\chi_1} \\ x: &\quad AzF_x = \frac{1-c\chi_3}{1-c\xi+c\chi_1} \\ AR: &\quad \frac{1}{c}r^{tc} = \xi(1+r^{tc}-\gamma_2) + \chi_1 - \chi_2 \\ AP: &\quad \frac{1}{c}r^{tc} = \xi(1+r^{tc}) - \chi_3 + \chi_4 \\ a': &\quad \frac{1}{c} = \beta\mathbb{E}_{z'|z}V_{a'}(a',z') + \xi\gamma_1 + \tau \end{aligned}$$

The envelope theorem is

$$V_a(a,z) = \frac{1}{c}(1+r)$$

That gives the Euler equation

$$\frac{1}{c} = \beta\mathbb{E}_{z'|z}\frac{1}{c'}(1+r') + \xi\gamma_1 + \tau \tag{B.4}$$

In addition, according to the Kuhn-Tucker condition, the Lagrangian multipliers and the

constraints have the following properties:

$$\xi \geq 0, \gamma_1 a' + \gamma_2 AR - AzF(k,l,x) - (1+r^{tc})(AR - AP) \geq 0,$$
$$\chi_1 \geq 0, AzF(k,l,x) - AR \geq 0,$$
$$\chi_2 \geq 0, AR \geq 0,$$
$$\chi_3 \geq 0, x \geq AR,$$
$$\chi_4 \geq 0, AP \geq 0,$$
$$\tau \geq 0, a' \geq 0,$$

with complementary slackness.

B.3 Proofs of propositions 1 and 2

Before proceeding to the proofs of the propositions, I first prove the monotonicity of the optimal policy function in a lemma. To do this, I rewrite the value function as

$$\begin{aligned} V(a,z) &= \max_{c,a'} u((1+r)a + \pi(z,a') - a') + \beta \int_{z'} V(a',z') d\lambda(z',z) \\ s.t. \quad & a' \geq 0. \end{aligned} \tag{B.5}$$

where given a',

$$\begin{aligned} \pi(z,a') &= \max_{k,l,x,AR,AP} Az\left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi\right)^\mu - (r+\delta)k - wl - x + r^{tc}(AR - AP) \\ s.t. \quad & Az\left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi\right)^\mu + (1+r^{tc})(AR - AP) \leq \gamma_1 a' + \gamma_2 AR, \\ & 0 \leq AR \leq Az\left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi\right)^\mu, \\ & 0 \leq AP \leq x. \end{aligned} \tag{B.6}$$

Essentially, I are breaking down this dynamic problem into two problems and proving the monotonicity property in each one. First, given a', the static production optimization problem is established in B.6, which gives the optimal profit function $\pi(z,a')$ for a given z.

Second, the dynamic problem of choosing next period wealth a', taken as given the function $\pi(z, a')$ as shown in B.5.

Lemma 1. *The value function $v(a, z)$ is supermodular in a and has increasing differences in (a, z). Given z, the policy functions $k(a, z)$, $l(a, z)$, $x(a, z)$, $AR(a, z)$, $-AP(a, z)$ and $a'(a, z)$ increase in a.*

Proof. First, I consider the optimization problem B.6: I intend to show that the optimal policy increase with a' for a given z using Theorem 2.8.1 from Topkis (1998).[22] It is easy to verify that the feasibility set increases strictly with a'; therefore I only need to show that inequality 2.8.1 from Topkis (1998) is satisfied.

Denote $W(k, l, x, AR, AP) = AzF(k, l, x) - (r + \delta)k - wl - x + (r^{tc}(AR - AP))$. Given any $\{k_1, l_1, x_1, AR_1, AP_1\}$ and $\{k_2, l_2, x_2, AR_2, AP_2\}$, I need to show that

$$\begin{aligned} & W(k_1, l_1, x_1, AR_1, AP_1) + W(k_2, l_2, x_1, AR_2, AP_2) \\ \leq \; & W(k_1 \wedge k_2, l_1 \wedge l_2, x_1 \wedge x_2, AR_1 \wedge AR_2, AP_1 \wedge AP_2) + \\ & W(k_1 \vee \; k_2, l_1 \vee \; l_2, x_1 \vee x_2, AR_1 \vee AR_2, AP_1 \vee AP_2), \end{aligned}$$

which reduces to

$$\begin{aligned} & zAF(k_1, l_1, x_1) + zAF(k_2, l_2, x_2) \\ \leq \; & zAF(k_1 \wedge k_2, l_1 \wedge l_2, x_1 \wedge x_2) + zAF(k_1 \vee k_2, l_1 \vee l_2, x_1 \vee x_2). \end{aligned}$$

This is straightforward to prove because function $F(\cdot, \cdot, \cdot)$ features strictly increasing differences in its inputs. Following Theorem 2.8.1 of Topkis (1998), I know that the optimal policy increases with a'.

For the next step, I move on to the optimization problem B.5. Following Proposition 2 in Hopenhayn and Prescott (1992), I will show the supermodularity of the value function $V(a, z)$ and policy function $a'(a, z)$ increase with a.

[22] Topkis, D. M. (1998). Submodularity and Complementarity. Princeton University Press.

I need to verify the following three conditions using equations B.5 and B.6. First, $u((1+r)a + \pi(z, a') - a')$ is supermodular in (a, a') and has increasing differences in (a, a') given any z. Second, the graph of the feasibility set $\{a'|a' \geq 0\}$ is a sublattice. Third, $\lambda(\cdot, z)$ is increasing in z with respect to the first-order stochastic dominance. The second and third conditions are straightforward in this case. I next need to show that the first condition holds, which is equivalent to showing $\frac{\partial^2 u}{\partial a \partial a'} \geq 0$.

$$\frac{\partial^2 u}{\partial a \partial a'} = u''(c)(1+r)(\pi_{a'}(z, a') - 1).$$

Since $u''(\cdot) < 0$, it is sufficient to show that $\underline{\pi_{a'}(z, a') \leq 1}$.

Write the the Lagrangian of the problem B.6 as follows:

$$\begin{aligned}
\mathcal{L} \quad = \quad & AzF(k, l, x) - (r+\delta)k - wl - x + r^{tc}(AR - AP) \\
& + \xi(\gamma_1 a' + \gamma_2 AR - AzF(k, l, x) - (1 + r^{tc})(AR - AP)) \\
& + \chi_1(AzF(k, l, x) - AR) + \chi_2 AR + \chi_3(x - AP) + \chi_4 AP.
\end{aligned}$$

Note that this is different from the Lagrangian of the full dynamic problem B.3. With a slight abuse of notation, I use the same $(\xi, \chi_1, \chi_2, \chi_3, \chi_4)$ to represent the Lagrangian multipliers in both problems.

The FOCs are:

$$k: \quad AzF_k = \frac{r+\delta}{1 - \xi + \chi_1} \tag{B.7}$$

$$l: \quad AzF_l = \frac{w}{1 - \xi + \chi_1} \tag{B.8}$$

$$x: \quad AzF_x = \frac{1 - \chi_3}{1 - \xi + \chi_1} \tag{B.9}$$

$$AR: \quad r^{tc} = \xi(1 + r^{tc} - \gamma_2) + \chi_1 - \chi_2 \tag{B.10}$$

$$AP: \quad r^{tc} = \xi(1 + r^{tc}) - \chi_3 + \chi_4 \tag{B.11}$$

There are two cases I need to investigate. The first one is if the borrowing constraint is

not binding ($\xi(a', z) = 0$) and the second one is if it is binding ($\xi(a', z) > 0$).

(1) $\xi = 0$

If $\xi = 0$, from equation B.10 and B.11, I know that $\chi_1 = r^{tc}$, $\chi_2 = 0$, $\chi_3 = 0$ and $\chi_4 = r^{tc}$. Therefore equation B.7, B.8 and B.9 become

$$\begin{aligned} k: &\quad AzF_k = \frac{r+\delta}{1+r^{tc}} \\ l: &\quad AzF_l = \frac{w}{1+r^{tc}} \\ x: &\quad AzF_x = \frac{1}{1+r^{tc}} \end{aligned}$$

Denote the solution to the above system of equations as k^*, l^*, x^* and the corresponding output $y^* = AzF(k^*, l^*, x^*)$. Because $x_1 > 0$ and $x_4 > 0$, the complementary slackness conditions imply that $AR = y^*$ and $AP = 0$. As a result, given z, $\pi(z, a')$ does not change with a', as a result, $\pi_{a'}(z, a') = 0$.

(2) $\xi > 0$

If ξ, I know that the borrowing constraint holds with equality. That is,

$$y + (1 + r^{tc})(AR - AP) = \gamma_1 a' + \gamma_2 AR \Longrightarrow y = \gamma_1 a' - (1 + r^{tc} - \gamma_2)AR + (1 + r^{tc})AP$$

By definition, $\pi(z, a') = y - (r + \delta)k - wl - x + r^{tc}(AR - AP)$. Replacing y using the above equation yields

$$\pi(z, a') = \gamma_1 a' - (1 - \gamma_2)AR + AP - (r + \delta)k - wl - x$$

Since I have shown that $\frac{dk}{da'} \geq 0, \frac{dl}{da'} \geq 0, \frac{dx}{da'} \geq 0$, $\frac{dAR}{da'} \geq 0$ and $\frac{dAP}{da'} \leq 0$, and because $\gamma_1 < 1$, I have $\pi_{a'}(z, a') < 1$. *Q.E.D.* $\square$

B.3.1 Proof of Proposition 1

Cut-off for financial constraint Given z, define set $\mathbf{U}^z = \{a|\xi(a,z) = 0\}$. I intend to show that the set $\mathbf{U}^z$ is in the following form $(\underline{a}, \infty)$.[23] To do this, I first show that $\mathbf{U}^z$ has the following property: if $a \in \mathbf{U}^z$ and $\hat{a} > a$, then $\hat{a} \in \mathbf{U}^z$.

Let $a \in \mathbf{U}^z$. According to the definition of $\mathbf{U}^z$, I know that $\xi(a,z) = 0$. The complementary slackness condition then implies that for entrepreneur (a,z), the working capital constraint is not binding,

$$AzF(k,l,x) + (1+r^{tc})(AR - AP) < \gamma_1 a' + \gamma_2 AR.$$

According to equation B.4 and B.4, $\xi = 0$ implies that $\chi_2 = 0$, $\chi_1 = \frac{1}{c}r^{tc}$, $\chi_3 = 0$ and $\chi_4 = \frac{1}{c}r^{tc}$. Taking the value of ξ, χ_1, χ_2 back into equations B.4, B.4 and B.4, I get

$$\begin{aligned} k: &\quad AzF_k = \frac{r+\delta}{1+r^{tc}}, \\ l: &\quad AzF_l = \frac{w}{1+r^{tc}}, \\ x: &\quad AzF_x = \frac{1}{1+r^{tc}}. \end{aligned}$$

Since production function F is decreasing return to scale, there exist optimal k, l and x that solve the above system of three equations. Denote the solution as k^*, l^* and x^*. Since $\chi_1 > 0$ and $\chi_4 > 0$, the complementary slackness condition implies that $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$.

Let $m = Az\left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi\right)^\mu - (r+\delta)k - wl - x$, and the budget constraint B.1 can be re-written as,

$$c + a' = (1+r)a + m.$$

It is clear that m is maximized when $k = k^*$, $l = l^*$, $x = x^*$, $AR = AzF(k^*, l^*, x^*)$ and

[23] This statement is equivalent to the first part of Proposition 1.

$AP = 0$. In other words, entrepreneurs will always choose $k = k^*$, $l = l^*$, $x = x^*$, $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$ if they are feasible under the working capital constraint (equation B.2).

Consider the entrepreneur with productivity z and wealth $\hat{a} > a$. According to Lemma 1, $a'(\hat{a}, z) \geq a'(a, z)$. Therefore, since $k = k^*$, $l = l^*$, $x = x^*$, $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$ are feasible for entrepreneur (a, z), they must be feasible for entrepreneur $(\hat{a}, z)$ as well. Following the above analysis, I know that entrepreneurs will choose $k = k^*$, $l = l^*$, $x = x^*$, $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$, and the working capital constraint holds with strict inequality. Using the complementary slackness condition, this implies that $\xi(\hat{a}, z) = 0$.

With the help of this property, I show that $\mathbf{U}^z$ is an interval. Suppose that it is not; then there exists $x < w < y$, such that $x, y \in \mathbf{U}^z$ but $w \notin \mathbf{U}^z$. This violates the property, since it means $x \in \mathbf{U}^z$, $w < x$, but $w \notin \mathbf{U}^z$. I can also show that $\mathbf{U}^z$ is unbounded from above. Suppose that it is not; then there exists $w \notin \mathbf{U}^z$ but $w > a$ for all $a \in \mathbf{U}^z$, which violates the property.

Cut-off for AR Define a set $\mathbf{H}^z = \{a | AR(a, z) > 0\}$. I show that $\mathbf{H}^z$ is in the form of $(\underline{a}, \infty)$. The proof is very similar. Essentially, I need to prove that the set $\mathbf{H}^z$ has the following property: if $a \in \mathbf{H}^z$ and $\hat{a} > a$, then $\hat{a} \in \mathbf{H}^z$. It is clear that this property holds since according to Lemma 1, $AR(a, z)$ is an increasing function in a. Therefore, for any $\hat{a} > a$, I have $AR(\hat{a}, z) \geq AR(a, z) > 0$.

Cut-off for AP Similarly, define a set $\mathbf{W}^z = \{a | AP(a, z) = 0\}$. I can show that $\mathbf{W}^z$ is in the form of $(\underline{a}, \infty)$. According to Lemma 1, $AP(a, z)$ is a decreasing function in a. Therefore for any $\hat{a} > a$, I have $0 \leq AP(\hat{a}, z) \leq AP(a, z) = 0$. As a result, $AP(\hat{a}, z) = 0$. *Q.E.D.*

B.3.2 Proof of Proposition 2

Proving this proposition is equivalent to showing that $\mathbf{U}^z \subseteq \mathbf{W}^z \subseteq \mathbf{H}^z$. I do it in two steps.

$\mathbf{U}^z \subseteq \mathbf{W}^z$ Take any $a \in \mathbf{U}^z$ and ξ be the Lagrangian multiplier associated with it; I know that $\xi = 0$ according to the definition of $\mathbf{U}^z$. According to equation B.4, if $\xi = 0$ then $\frac{1}{c(a,z)} r^{tc} = \chi_4(a,z) - \chi_3(a,z)$. Since $\frac{1}{c(a,z)} r^{tc} > 0$, it has to be the case that $\chi_4(a,z) = \frac{1}{c(a,z)} r^{tc}$ and $\chi_3(a,z) = 0$. Apply the complementary slackness condition, I know $AP(a,z) = 0$, which means $a \in \mathbf{W}^z$.

$\mathbf{W}^z \subseteq \mathbf{H}^z$ For any $a \in \mathbf{W}^z$, I know $AP(a,z) = 0$, thus the complementary slackness condition implies that $\chi_4(a,z) > 0$ and $\chi_3(a,z) = 0$. Therefore equation B.4 implies $\frac{1}{c} r^{tc} > \xi(1 + r^{tc})$. As a result, $\frac{1}{c} r^{tc} > \xi(1 + r^{tc} - \gamma)$ because $\xi(a,z) \geq 0$. Take $\frac{1}{c} r^{tc} > \xi(1 + r^{tc} - \gamma)$ back to equation B.4, I get $\chi_1(a,z) > 0$ and $\chi_2(a,z) = 0$. The complementary slackness condition implies that $AR(a,z) > 0$, which means $a \in \mathbf{H}^z$. *Q.E.D.*

B.4 Equilibrium definition of the counterfactual economy

The stationary equilibrium of the counterfactual economy without trade credit is defined as follows:

Definition. The recursive competitive equilibrium consists of interest rate of rental capital r, wage rate w; value function of the entrepreneurs $V(a,z)$; policy functions $c(a,z)$, $k(a,z)$, $l(a,z)$, $x(a,z)$, and $a'(a,z)$; consumption and hours of the workers (c^h, h); and the CDF of the stationary distribution $\Phi(a,z)$, such that

1. Given prices, the value functions and policy functions solve the entrepreneurs' problem.

$$\begin{aligned} V(a,z) &= \max_{c,k,l,x,AR,AP,a'} log(c) + \beta \mathbb{E}_{z'} V(a',z'), \\ s.t. \quad & c + a' = (1+r)a + Az((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi)^\mu - (r+\delta)k - wl - x, \\ & Az((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi)^\mu \leq \gamma_1 a', \ a' \geq 0. \end{aligned}$$

2. Given prices, the consumption and hours of the workers solve the workers' problem.

3. Labor market clears

$$\int l(a,z)d\Phi(a,z) = N \cdot h.$$

4. Rental capital market clears

$$\int k(a,z)d\Phi(a,z) = \int a d\Phi(a,z).$$

5. Goods market clear

$$\int y(a,z)d\Phi(a,z) = N \cdot c^h + \int [c(a,z) + a'(a,z) - a + x(a,z)]\Phi(a,z).$$

C Quantitative exercises appendix

C.1 Algorithms

In this section, I describe the algorithms for computing the benchmark model. The algorithms to compute the counterfactual model are very similar to the benchmark model, only with different sets of FOCs, budget constraints, and working capital constraints. Hence they are omitted here.

C.1.1 Stationary equilibrium

- Guess equilibrium prices r, w, r^{tc}.
- Given the prices, solve the household problem.
- Given the prices, solve the entrepreneurs problem as follows:
  - Discretize the state space.
  - Guess policy function $c(a, z)$.
  - For each (a, z), assume that the entrepreneur is unconstrained, i.e., $\mu(a, z) = 0$. Solve for the system of equations that consists of FOCs and budget constraint.
  - Check whether the working capital constraint is satisfied with the solution to the above system of equations.
  - If the working capital constraint is not satisfied, it means that $\mu(a, z) > 0$ and working capital constraint holds with equality. Solve the system of equations that consists of FOCs, budget constraint, and working capital constraint (with equality).
  - Use the Euler equation to update the policy function $c(a, z)$ until it converges.
- Given any arbitrary distribution of (a, z), iterate using the policy functions derived above until a stationary distribution is reached.

- Generate the aggregate statistics of the three markets: capital, labor, and trade credit market.
- Update (r, w, r^{tc}) until the markets clear simultaneously.

C.1.2 Transitional dynamics

To compute the transitional dynamics of the economy, I consider a transition path of $T = 100$ periods. The economy is at the initial stationary equilibrium level in period $t = 1$, and I assume that it converges back to the initial stationary equilibrium at period $t = T$.

- Guess a sequence of prices $\{r_t, w_t, r_t^{tc}\}_{t=2}^{T-1}$.
- Backward induction. For each $t = T-1, T-2, ..., 2$,
  - Discretize the state space.
  - Given prices, solve the household problem for period t.
  - Given prices, solve the entrepreneurs' policy functions for period t.
    1. Guess $c_t(a, z)\mu_t(a, z) = 0$, solve the system of equations that consists of FOCs of period t, budget constraint, and Euler equations (with the next period policy function $c_{t+1}(a, z)$ known).
    2. Check whether the working capital constraint is satisfied under the above solution.
    3. If the working capital is not satisfied, $c_t(a, z)\mu_t(a, z) > 0$ and the working capital constraint holds with equality. Solve the system of equations that consists of FOCs of period t, budget constraint, Euler equations (with the next period policy function $c_{t+1}(a, z)$ known), and working capital constraint with equality.
- Forward induction. The first period stationary distribution $\Phi_1(a, z)$ is set to be the stationary equilibrium distribution. Using the policy functions for period $t = 2, ..., T-1$, compute the distribution along the transition path $\Phi_t(a, z)$.

- Generate aggregate statistics for the four markets in every period $t = 2, ..., T-1$ using the policy functions and the distributions.
- Update $\{r_t, w_t, r_t^{tc}\}_{t=2}^{T-1}$ until the four markets clear simultaneously in each period $t = 2, ..., T-1$.

C.2 The shadow value of liquidity function

I begin by looking at how the shadow value of liquidity ξ varies with wealth a for a given productivity level. Figure C.4 shows that ξ decreases in a, indicating that producers are less constrained with higher wealth. Notably, ξ is not strictly monotone; the function is divided into six segments by three values of ξ: $\{0, r^{tc}, r^{tc}/(1-\gamma_2)\}$. Each segment is associated with different value ranges for AR and AP.

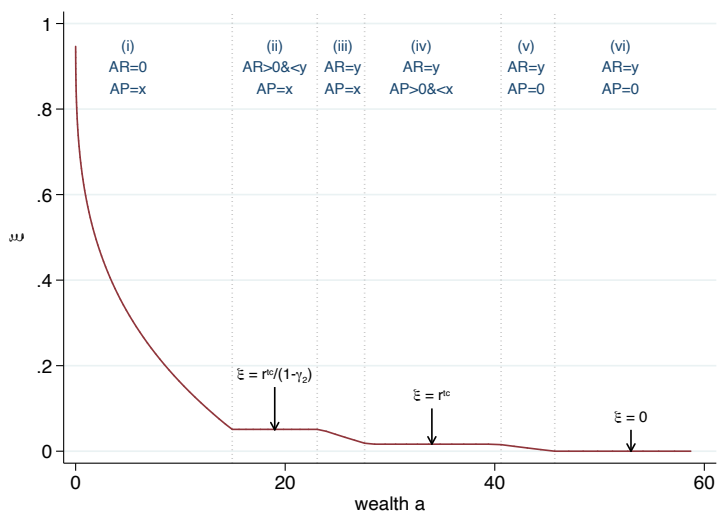


Figure C.4: Shadow value of liquidity and its role in determining trade credit choices

Notes: This figure plots the liquidity value ξ as a function of wealth a for a given z. The function ξ decreases in a, but not strictly decreasing. There are three regions where ξ is constant, marked by $\xi \in \{r^{tc}/(1-\gamma_2), r^{tc}, 0\}$. As such, the function ξ is divided into six segments, each associated with a different value range for AR and AP.

C.3 Policy functions

The policy functions of AR and AP as shown in figure 6 indicates that some entrepreneurs choose not to lend or borrow trade credit (AR = 0 or AP = 0). In the data, however, it is very rare for firms to have zero AR or AP on their balance sheet. In this section, I show that this discrepancy can be solved by recognizing that measured AR and AP is a snapshot of firms' activities and they likely reflect the averages of AR and AP over several sales and purchases that overlapped in time.

In figure C.5, I plot AR/y and AP/x averaged over three sales/production cycles. More specifically, I track each entrepreneurs over three periods and use average $\{\text{AR}, \text{AP}, x, y, a\}$ to plot this figure. The shapes of AR/y and AP/x can more closely resemble the data patterns. In particular, they no longer take the corner values as before.

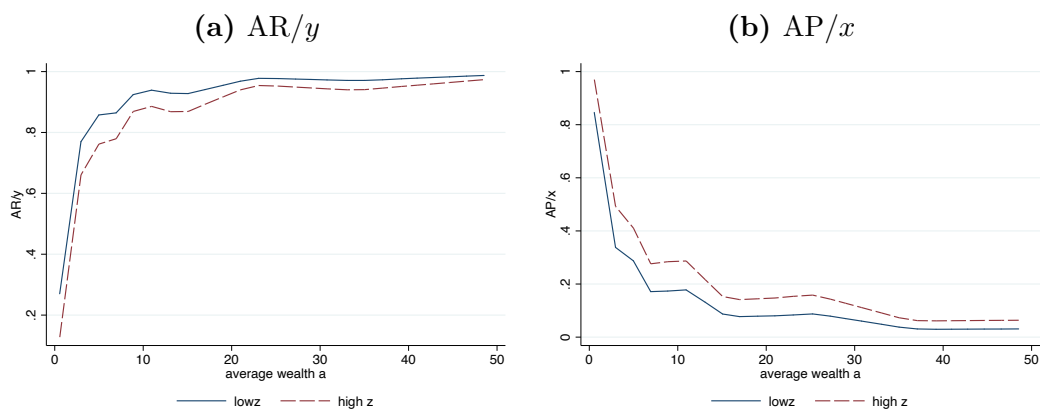


Figure C.5: Trade credit choice, average over three periods

Notes: This figure displays trade credit choices as a function of wealth a for given low and high values of z. Panel (a) shows the average AR/average output over three periods. Panel (b) shows the average AP/average input costs over three periods.

In the data, firms might make multiple sales and purchases that overlap in time; therefore, the observed AR and AP may reflect the average of these sales and purchases. In this exercise, I consider the case where AR, AP, and sales reflect the averages of three cycles of sales/production. Although in the model, I do not explicitly model the *overlap* of these sales, this exercise conveys the simple idea that, in the presence of idiosyncratic productivity

randomness (shocks to z), taking averages smooths out the policy functions; hence they can better mimic the data.

C.4 Aggregate dynamics in the benchmark economy with fixed trade credit interest rate versus the counterfactual economy

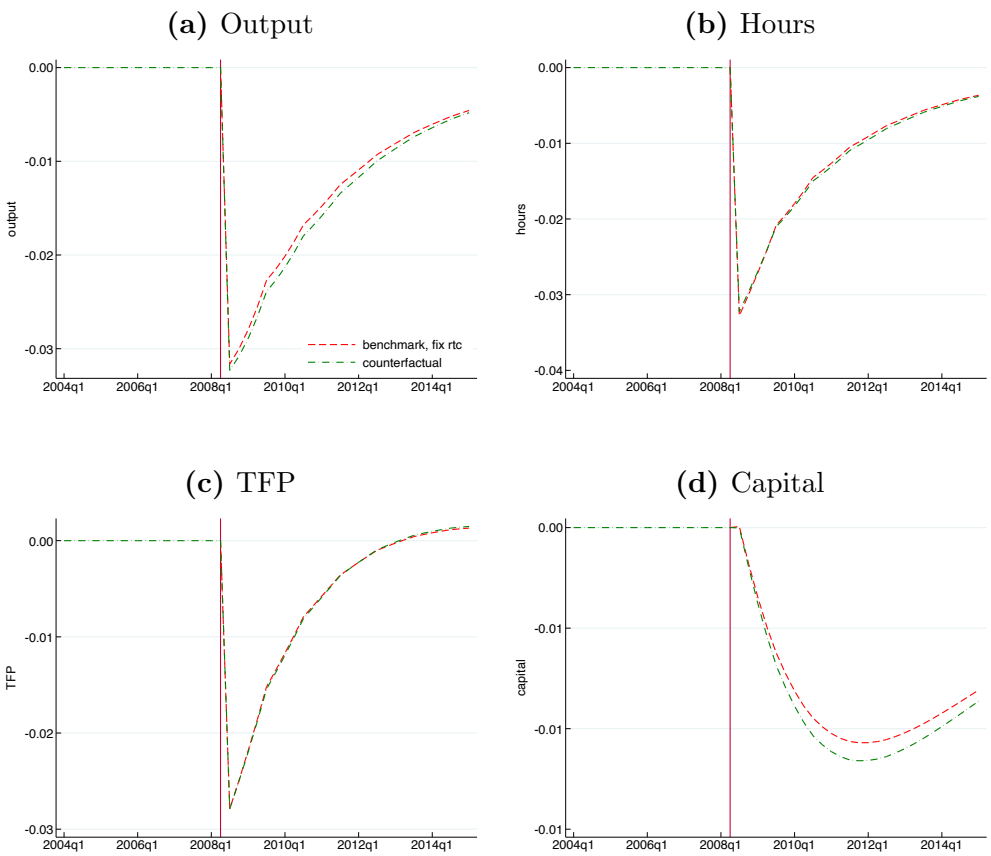


Figure C.6: Dynamics of the aggregate variables: benchmark with fixed trade credit interest rate vs. counterfactual

Notes: The figures show the changes in the aggregate economy in terms of output, hours, aggregate TFP, and capital stock after the financial crisis. All lines are normalized to 0 at the beginning of the crisis. Each line in the figure represents a model economy: benchmark economy with fixed trade credit interest rate (red), and counterfactual economy without trade credit (green).