Allocative Efficiency and the Productivity Slowdown*

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December 11, 2020

Abstract

This paper evaluates the contribution of cross-sector allocative efficiency to the slowdown in productivity growth during the 1970s and the 2000s in the US. We extend the framework of Oberfield (2013) to derive sufficient statistics for allocative efficiency and decompose aggregate productivity growth in 1) a multi-sector value-added economy, and 2) an input-output economy à la Jones (2013). We find that the lack of improvement in allocative efficiency can explain approximately two-thirds of the US's productivity slowdown. The allocation of capital was the main driver behind the overall movement in allocative efficiency, both in the long run and during the two slowdown episodes. Manufacturing and service sectors both contributed to the deterioration of allocative efficiency in 1970s whereas the decline in allocative efficiency was more concentrated in the manufacturing sectors during the 2000s.

Keywords: Allocative efficiency; misallocation; productivity slowdown; input-output linkages.

JEL Code: O47; E23; D57; C67.

*The views expressed in this paper are solely those of the author and may differ from the official Bank of Canada views. Errors are our own. We thank Susanto Basu, Gabriela Galassi, Oleksiy Kryvtsov, Markus Poschke, B. Ravikumar, Diego Restuccia, and Faisal Sohail for helpful comments. Shao: Department of International Economic Analysis, Bank of Canada, 234 Wellington Street, Ottawa, ON, Canada, K1A 0G9 (e-mail: lin.j.shao@gmail.com). Tang: Institute for Advanced Research, Shanghai University of Finance and Economics; Key Laboratory of Mathematical Economics (SUFE), Ministry of Education, 777 Guoding Road, Shanghai, China, 200433 (e-mail: tang.rongsheng@sufe.edu.cn). Tang is grateful for the financial supports from the National Natural Science Foundation of China (No.71803112).

1 Introduction

Growth in real output per worker in the US slowed down significantly in the 1970s and the 2000s (see Figure 1). The slowdown in productivity growth was among the most significant macroeconomic developments in the past few decades and has captured much attention among academic researchers and policymakers. This paper evaluates the role of allocative efficiency—the efficiency with which factors of production are allocated across sectors—in explaining these two episodes of slow productivity growth. To do so, we develop a framework to decompose the aggregate productivity growth into changes in allocative efficiency and a residual term that we interpret as changes in fundamental technology. We show that allocative efficiency—or more precisely, the lack of improvement in allocative efficiency—was the common factor behind both episodes of slow productivity growth.

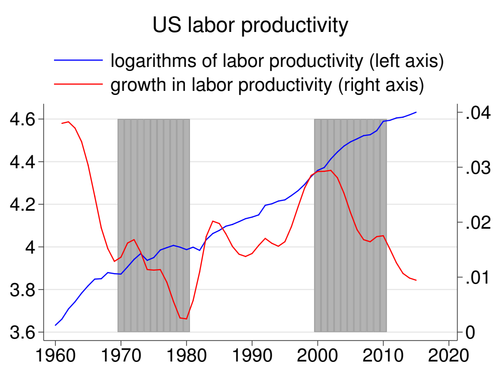


Figure 1: **Labor productivity in the US**

Source: BLS and FRED.
Notes: This figure plots the logarithms and growth of real output per worker in the United States business sector. The growth rate is computed as the hp-filtered log difference in real output per worker. The growth of real output per hour follows a very similar trend as real output per worker (see Figure 1 of Vandenbroucke, 2019). The gray bars indicate the two productivity slowdown episodes.

The framework builds upon Oberfield (2013) and features 1) a multi-sector value-added economy, and 2) an input-output economy à la Jones (2013). We characterize the planner's optimal allocation problem and derive sufficient statistics for measuring allocative efficiency. Intuitively, the sufficient statistics of allocative efficiency captures the deviation of the cross-sector allocation of production factors—capital, labor, and intermediate inputs—in the data from that under the optimal allocation. The empirical exercises rely on the sector-level data in KLEMS and the input-output table collected by World Input-Output Table (WIOT). Empirically, the allocative efficiency played a quantitatively important role in explaining the productivity slowdown in both decades.

We find that allocative efficiency improved gradually over time in the US. Throughout 1960-2007, allocative efficiency grew by 16-18 percent and contributed to approximately 18-20 percent of aggregate productivity growth. In contrast, the contribution of allocative efficiency to aggregate productivity growth was at most 1 percent during the 1970s and 3 percent during the 2000s. By comparing productivity growth in the data and that under the optimal allocation, we show that the lack of improvement in allocative efficiency in these two decades can explain approximately two-thirds of the slowdown in productivity growth.

Capital, not labor or intermediate inputs, was the main driver behind movements in allocative efficiency, both in the long run and during the two slowdown episodes. Over time, the resources allocated to the manufacturing and service sectors moved closer to the optimal level. During the 1970, manufacturing and service sectors both experienced a decline in allocative efficiency, with service sectors allocation deteriorating and recovering slightly earlier than the manufacturing sectors. In contrast, the decline in allocative efficiency was more concentrated in the manufacturing sectors in the 2000s.

We introduce input-output linkages into the model with the prior that they might change the measured allocative efficiency. Empirically, we find that the *level* of allocative efficiency is indeed lower in the input-output economy than the value-added economy. However, the *changes* of allocative efficiency are almost identical in these two economies for the period when the input-output information is available. As a result, input-output linkages do not alter the decomposition of the aggregate productivity growth significantly because it is the change, not the level, that matters for growth.

A challenge in measuring allocative efficiency is the assumptions we make to obtain the output elasticities in the production functions. We employ two specifications. Specification

1 assumes that the factor shares are undistorted, corresponding to the form of sectoral level wedges as in Jones (2013). In specification 2, we assume that factor shares can be distorted in each year, but they are, on average, undistorted over time (Oberfield, 2013). Our results are robust to these different specifications.

We contribute to the literature studying productivity slowdown in the 1970s by showing an integral role for allocative efficiency. Existing literature on this topic focuses on the rise in the oil price (Jorgenson, 1988), measurement errors (Baily and Gordon, 1988), information technology (Greenwood and Yorukoglu, 1997 and Hornstein and Krusell, 1996), and demography (Feyrer, 2007 and Vandenbroucke, 2019). We show in this paper that allocative efficiency played a quantitatively important role, which to our knowledge has not been discussed before in the context of the 1970s productivity slowdown.[1]

Measurement errors and the slowdown in technological progress are also cited as contributors to the post-2000 slowdown in productivity growth. Some researches argue that the official statistics overstates the productivity slowdown in the 2000s because of the increasing mismeasurement in productivity gain from IT-related goods and services. Byrne, Fernald and Reinsdorf (2016)and Syverson (2017) evaluate this argument and find that mismeasurement cannot explain a substantial part of the productivity slowdown. Cette, Fernald and Mojon (2016) document that the easing in utilization and adoption of IT technology contributed to the slowdown in productivity growth before the Great Recession in the US. Aum, Lee and Shin (2018) attribute the aggregate productivity slowdown to more dispersed sector-level growth rates in the presence of cross-sector complementarity. Ramey (2020) argues that the US economy is currently going through a technology lull, i.e. a temporary state of slow technological progress. Other researches such as Bloom et al. (2020) and Gordon (2016) suggest that the slow technology progress might be a permanent state. Lastly, Decker et al. (2017) find that the dampened growth in allocative efficiency of labor across firms contributed significantly to the productivity slowdown from the late 1990s to the mid-2000s.

[1] See Vandenbroucke (2019) for a recent summary of the literature on the 1970s productivity slowdown.

Using a different dataset and methodology, our results also attribute much of the slowdown in productivity growth to allocation.

This paper develops a tractable framework to decompose aggregate productivity growth, making use of the sector-level data sets such as KLEMS and WIOT. The framework builds upon the literature that measures loss from misallocation without input-output linkages (Hsieh and Klenow, 2009) and with linkages (Jones, 2011, Jones, 2013, Leal, 2015, Osotimehin and Popov, 2020, Liu, 2019 and Hang, Krishna and Tang, 2020). It is also related to the research that use firm-level data to study the impact of misallocation on aggregate productivity growth (Gopinath et al., 2017 and Calligaris et al., 2018) or fluctuations (Oberfield, 2013 and Osotimehin, 2019). Basu and Fernald (2002) and Baqaee and Farhi, 2020 also study the impact of allocative efficiency on aggregate productivity growth using a decomposition framework. But their frameworks are based on different notions of allocative efficiency, and thus the results are not directly comparable to ours (see discussions in Baqaee and Farhi, 2020).

The structure of the paper is as follows: in section 2, we characterize the optimal allocation and the decomposition framework. Section 3 and 4 apply the framework to study the slowdown of US productivity growth. Section 5 presents additional discussions on the methodology and the results. Section 6 concludes.

2 Theoretical framework

This section presents the theoretical framework in three steps. First, we characterize the optimal allocation across sectors as a planner's problem. Second, we define allocative efficiency and derive sufficient statistics to measure it using the optimal allocation from step 1. Last, we show a simple framework that decomposes aggregate labor productivity growth using the measured allocative efficiency from step 2. In section 2.1 we consider an economy without input-output linkages. We then turn to the economy with input-output linkages in section 2.2. In the literature, these are also often referred to as the value-added economy and the input-output economy, respectively.

2.1 Value-added economy

There are N sectors in the economy ($i = \{1, ..., N\}$). In year t, each sector produces a good $Y_{i,t}$ using capital, labor and a Cobb-Douglas production function

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}},$$

where $A_{i,t}$ is the sectoral productivity.

There is one final good Y_t, which is produced by aggregating all sectoral goods, such that

$$Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}},$$

in which $\sum_i \theta_{i,t} = 1$.

2.1.1 Planner's problem

The planner's problem is to allocate aggregate capital K_t and labor L_t into the N sectors to maximize the output of final good Y_t, such that,

$$\max \; Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}}, s.t. \; Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}}, \; \sum_i K_{i,t} = K_t, \; \sum_i L_{i,t} = L_t$$

The following proposition characterizes the optimal allocation across sectors and the optimal output.

Proposition 1. *The optimal allocation of capital and labor in this economy is such that* $K_{i,t}^* = \chi_{i,t}^{k*} K_t$ *and* $L_{i,t}^* = \chi_{i,t}^{l*} L_t$*, where* $\chi_{i,t}^{k*} = \frac{\theta_{i,t}\alpha_{i,t}}{\sum_i \theta_{i,t}\alpha_{i,t}}$ *and* $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\alpha_{i.t})}{\sum_i \theta_{i,t}(1-\alpha_{i,t})}$.

Proof. See Appendix B.1. □

Under the optimal allocation, aggregate capital and labor are allocated to each sector according to the optimal sectoral shares $\chi^{k*}_{i,t}$ and $\chi^{l*}_{i,t}$. Intuitively, the optimal sectoral shares reflect the relative importance of sector i's capital and labor in the production of the final good ($\alpha_i\theta_i$ and $(1-\alpha_i)\theta_i$, respectively).

Allocative efficiency We define allocative efficiency $\mathbf{E}_t$ as the ratio between output in the data (Y_t) and output under the optimal allocation (Y^*_t),

$$\mathbf{E}_t = \frac{Y_t}{Y^*_t},$$

It can be shown, using Proposition 1, that

$$\mathbf{E}_t = \prod_{i=1}^{N}[(\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}})^{\alpha_{i,t}}(\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}})^{1-\alpha_{i,t}}]^{\theta_{i,t}}, \tag{1}$$

where $\chi^k_{i,t} = \frac{K_{i,t}}{K_t}$ and $\chi^l_{i,t} = \frac{L_{i,t}}{L_t}$ are the sector i's capital and labor as a share of aggregate K_t and L_t in the data, respectively (see Appendix B.1.1 for details). Intuitively, $(\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}})^{\alpha_{i,t}}(\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}})^{1-\alpha_{i,t}}$ measures sector i's allocative efficiency, which is the deviation of the observed allocation in the data from the optimal allocation in sector i. Aggregate allocative efficiency $\mathbf{E}_t$ is then simply the weighted geometric average of sectoral allocative efficiency with sectoral weights θ_i.

2.2 Input-output economy

In contrast to the value-added economy, in the input-output economy, each sector $i \in \{1, ..., N\}$ produces good $Q_{i,t}$ using capital, labor, domestic and imported intermediate goods, such that

$$Q_{i,t} = A_{i,t}(K^{\alpha_{i,t}}_{i,t} L^{1-\alpha_{i,t}}_{i,t})^{1-\sigma_{i,t}-\lambda_{i,t}}(\prod_{j=1}^{N} d^{\sigma_{ij,t}}_{ij,t})(\prod_{j=1}^{N} m^{\lambda_{ij,t}}_{ij,t}),$$

where $d_{ij,t}$ is the domestic intermediate good j used by sector i, $m_{ij,t}$ is the imported intermediate good j used by sector i, $\sigma_{i,t} = \sum_{j=1}^{N} \sigma_{ij,t}$, and $\lambda_{i,t} = \sum_{j=1}^{N} \lambda_{ij,t}$.[2]

There is a final good in the economy, produced by aggregating over the sectoral goods,

$$Y_t = \prod_i Y_{i,t}^{\theta_{i,t}},$$

where $\sum_{i=1}^{N} \theta_{i,t} = 1$.

The resource constraint on the sectoral good i therefore can be written as

$$Q_{i,t} = Y_{i,t} + \sum_{j=1}^{N} d_{ji,t},$$

and the total expenditure on imported goods is

$$X_t = \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{P}_{j,t} m_{ij,t},$$

where $\bar{P}_{j,t}$ is the price of imported intermediate good j relative to the final good.

2.2.1 Planner's problem

The planner's problem is to allocate aggregate capital K_t, aggregate labor L_t, sectoral output $Q_{i,t}$ and choose imported intermediate good $m_{ij,t}$ such that the aggregate output net of imports $(Y - X)$ is maximized,

$$\begin{aligned} \max_{\{K_{i,t}, L_{i,t}, d_{ij,t}, m_{ij,t}\}_{i,j=1}^{N}} \quad & Y_t - X_t = \prod_i Y_{i,t}^{\theta_{i,t}} - \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{P}_{j,t} m_{ij,t} \\ s.t. \quad & Q_{i,t} = A_{i,t} (K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}} (\prod_{j=1}^{N} d_{ij,t}^{\sigma_{ij,t}}) (\prod_{j=1}^{N} m_{ij,t}^{\lambda_{ij,t}}), \\ & Q_{i,t} = Y_{i,t} + \sum_{j=1}^{N} d_{ji,t}, \ \sum_i K_{i,t} = K_t, \ \sum_i L_{i,t} = L_t. \end{aligned}$$

[2] We also studied an input-output economy without international trade and the results are quantitatively similar to the input-output economy with trade.

The optimal allocation is characterized by the following proposition,

Proposition 2. *The optimal allocation of capital, labor and intermediate goods in the economy can be characterized using optimal sectoral shares* ($\chi^{k*}_{i,t}$, $\chi^{l*}_{i,t}$, $\gamma^{*}_{ij,t}$, $\chi^{y*}_{i,t}$)*, such that* $K^*_{i,t} = \chi^{k*}_{i,t} K_t$, $L^*_{i,t} = \chi^{l*}_{i,t} L_t$, $d^*_{ij,t} = \gamma^*_{ij,t} Q^*_{j,t}$, $Y^*_{j,t} = \chi^{y*}_{j,t} Q^*_{j,t}$, *and* $m^*_{ij,t} = (\frac{\theta_{i,t}\lambda_{ij,t}}{\chi^{y*}_{i,t}})\frac{Y^*_t}{P_{j,t}}$ *such that*

1. $\chi^{k*}_{i,t} = \frac{\theta_{i,t}\alpha_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})}{1-\sum_j \gamma^*_{ji,t}} / \sum_s \frac{\theta_{s,t}\alpha_{s,t}(1-\sigma_{s,t}-\lambda_{s,t})}{1-\sum_j \gamma^*_{js,t}}$, $\forall i \in \{1, ..., N\}$.
2. $\chi^{l*}_{i,t} = \frac{\theta_{i,t}(1-\alpha_{i,t})(1-\sigma_{i,t}-\lambda_{i,t})}{1-\sum_{j,t} \gamma^*_{ji,t}} / \sum_s \frac{\theta_{s,t}(1-\alpha_{s,t})(1-\sigma_{s,t}-\lambda_{s,t})}{1-\sum_j \gamma^*_{js,t}}$, $\forall i \in \{1, ..., N\}$.
3. $\{\chi^{y*}_{i,t}\}_{i=1}^N$ *solve the system of equations*

$$\frac{1}{\chi^y_{i,t}} = 1 + \frac{1}{\theta_{i,t}} \sum_s (\frac{\theta_{s,t}}{\chi^y_{s,t}} \sigma_{si,t}), i \in \{1, ..., N\}, \quad (2)$$

and

$$\gamma^*_{ij,t} = \frac{\theta_{i,t}\chi^{y*}_{j,t}}{\theta_{j,t}\chi^{y*}_{i,t}} \sigma_{ij,t}. \quad (3)$$

4. $\{Q^*_{i,t}\}_{i=1}^N$ *solve for the system of equations*

$$Q_{i,t} = \chi_{Qi,t} (\prod_{s=1}^N Q_{s,t}^{\sigma_{is,t}+\lambda_{i,t}\theta_{s,t}}), i \in \{1, ..., N\},$$

where $\chi_{Qi,t} = A_{i,t}[(\chi^{k*}_{i,t} K_t)^{\alpha_{i,t}} (\chi^{l*}_{i,t} L_t)^{1-\alpha_{i,t}}]^{1-\sigma_{i,t}-\lambda_{i,t}} (\prod_{j=1}^N \gamma_{ij,t}^{*\sigma_{ij,t}}) [\theta_{i,t} \prod_s (\frac{\chi^{y*}_{s,t}}{\chi^{y*}_{i,t}})^{\theta_{s,t}}]^{\lambda_{i,t}} \prod_{j=1}^N (\frac{\lambda_{ij,t}}{P_{j,t}})^{\lambda_{ij,t}}$.

Proof. See Appendix B.2. Note that the optimal shares only depend on the output elasticities in the production functions. □

Allocative efficiency We define the allocative efficiency as the ratio between the output net of imports in the data and that under the optimal allocation, such that

$$\mathbf{E}_t = \frac{Y_t - X_t}{Y^*_t - X^*_t}.$$

It can be shown using proposition 2 that $\mathbf{E}_t$ can be written as a product of allocative efficiency of capital, labor, domestic and imported intermediate goods, and intermediate goods used for final good production, such that

$$\mathbf{E}_t = E_t^{kl} \cdot E_t^{d} \cdot E_t^{m} \cdot E_t^{y}, \tag{4}$$

- $E_t^{kl} = \prod_{i=1}^{N}(((\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}}(\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t}C_{ni,t}}$ is the allocative efficiency of capital and labor.
- $E_t^{d} = \prod_{i=1}^{N}(\prod_{j=1}^{N}(\frac{\gamma_{ij,t}}{\gamma_{ij,t}^*})^{\sigma_{ij,t}})^{\sum_n \theta_{n,t}C_{ni,t}}$ is the allocative efficiency of domestic intermediate goods.
- $E_t^{m} = \frac{1-\sum_{n=1}^{N}\frac{\theta_{n,t}\lambda_{n,t}}{\chi_{n,t}^y}}{1-\sum_{n=1}^{N}\frac{\theta_{n,t}\lambda_{n,t}}{\chi_{n,t}^{y*}}}$ is the allocative efficiency of imported intermediate goods.
- $E_t^{y} = \prod_{n=1}^{N}(\frac{\chi_{n,t}^y}{\chi_{n,t}^{y*}})^{\theta_{n,t}} \prod_{i=1}^{N}(\frac{\prod_s(\frac{\chi_{s,t}^y}{\chi_{i,t}^y})^{\theta_{s,t}}}{\prod_s(\frac{\chi_{s,t}^{y*}}{\chi_{i,t}^{y*}})^{\theta_{s,t}}})^{\lambda_{i,t}\sum_n(\theta_{n,t}C_{ni,t})}$ is the allocative efficiency of intermediate goods used in the final goods production.

where C_t is the $N \times N$ matrix, such that $C_t = (I - \Omega_t)^{-1}$ and $\Omega_t(i,j) = \sigma_{ij,t} + \lambda_{i,t}\theta_{j,t}$. Without international trade, i.e. $\lambda_{i,t} = 0$, C_t is simply the Leontief inverse matrix. In the above equation, $(\chi_{i,t}^{k*},\ \chi_{i,t}^{l*},\ \gamma_{ij,t}^*,\ \chi_{i,t}^{y*})$ are the optimal allocation of capital, labor and intermediate inputs across sectors and $(\chi_{i,t}^{k},\ \chi_{i,t}^{l},\ \gamma_{ij,t},\ \chi_{i,t}^{y})$ are their data analogs.[3]

2.3 Decomposition of aggregate productivity in the data

This section uses the theoretical results in the previous two sections and shows the decomposition of the aggregate labor productivity in the data.

Proposition 3. *Aggregate labor productivity measured in the data LP_t can be decomposed into 1) allocative efficiency $\mathbf{E_t}$ and 2) aggregate labor productivity under optimal allocation*

[3] See Appendix B.2.1 for details of the algebra. In deriving equation 4, we assume that the expenditure shares of imported intermediate goods are undistorted.

LP_t^*, *such that*

$$LP_t = LP_t^* \mathbf{E_t} \quad (5)$$

$$\triangle log LP_t = \triangle log LP_t^* + \triangle log \mathbf{E_t}. \quad (6)$$

Proof. The proof can be found in Appendix B.3. □

Equation 5 and 6 are the decomposition of the level and the growth rate of labor productivity. The focus of our paper is to study the contribution of changes in allocative efficiency ($\triangle log \mathbf{E_t}$) to aggregate productivity growth ($\triangle log LP_t$). It is clear from equation 5 that the allocative efficiency captures the distance of the data (LP) to the production possibility frontier (LP^*). Both LP and $\mathbf{E}_t$ can be measured from data directly. Therefore LP^* can be computed as a residual term using equation 5 and can be interpreted as the fundamental technology.

3 Application to the US data

In this section, we discuss the data sets used in the paper (section 3.1) as well as the empirical strategies to back out the cross-sector allocation in the data (section 3.2) and the output elasticities in the production functions (section 3.3).

3.1 Data description

We use the 2013 version of the KLEMS dataset and the 2013 version of the World Input-Output Table (WIOT). The 2013 version of KLEMS and WIOT are both based on the ISIC Rev. 3 classification thus allows a straightforward mapping between the two datasets. We restrict our analysis to $N = 28$ private sectors in the economy (marked red in Table 1).

The US KLEMS dataset covers the period of 1947-2010, while the input-output table covers 1995-2011, thus restricting the analysis with input-output linkages to the period of

1995-2010. The analysis without input-output linkages spans a longer period, allowing us to study the slowdown in the US productivity growth during the 1970s.

Table 1: List of sectors

Sectors
AtB Agriculture hunting forestry and fishing
C Mining and quarrying
D Manufacturing
15t16 Food products beverages and tobacco
17t19 Textiles textile products leather and footwear
20 Wood and products of wood and cork
21t22 Pulp paper paper products printing and publishing
23 Coke refined petroleum products and nuclear fuel
24 Chemicals and chemical products
25 Rubber and plastics products
26 Other non-metallic mineral products
27t28 Basic metals and fabricated metal products
29 Machinery nec
30t33 Electrical and optical equipment
34t35 Transport equipment
36t37 Manufacturing nec; recycling
E Electricity gas and water supply
F Construction
G Wholesale and retail trade
50 Wholesale trade and commission trade except of motor vehicles and motorcycles
51 Sale maintenance and repair of motor vehicles and motorcycles; retail sale of fuel
52 Retail trade except of motor vehicles and motorcycles; repair of household goods
H Hotels and restaurants
I Transport and storage and communication
60t63 Transport and storage
64 Post and telecommunications
J Financial intermediation
K Real estate, renting and business activities
70 Real estate activities
71t74 Renting of m&eq and other business activities
M Education
N Health and social work

The data has several limitations. First, WIOT reports the expenditure (nominal value) of intermediate inputs. But there is no measure of the quantity of the intermediate inputs. In section 5.2, we explore the consequences of the missing quantity data for measuring allocative efficiency. Second, KLEMS does not distinguish between profit (markups) and capital income, i.e., capital compensation + labor compensation = value added output for each sector in KLEMS. Therefore in our analysis, we abstract from markups. Third, we use real capital stock and the number of workers to measure capital and labor inputs. In KLEMS, real capital stock and the number of workers are taken from national accounts. Compared with the other measures constructed from survey data, they arguably suffer less from measurement errors.

Below we list all the variables used in the empirical exercise. For each variable, we distinguish whether it is the nominal value ($) or quantity.

KLEMS

- Sector-level value-added and gross output ($).
- Sector-level capital and labor compensation, and cost of intermediate goods ($).
- Sector-level real capital stock, and the number of workers (quantity).[4]

WIOT

- Sector i's use of domestic sector j good ($).
- Sector i's use of foreign sector j good ($)
- Sector i good used in the final good production ($).

3.2 Cross-sector allocation in the data

To calculate $\mathbf{E}_t$,we first need to compute the allocation of capital, labor and intermediate inputs across sectors in the data $(\chi^k_{i,t}, \chi^l_{i,t}, \chi^y_{i,t}, \gamma_{ij,t})$. Ideally, we would like to use inputs measured in quantities to calculate the allocation across sectors. We are able to do so for capital and labor, such that $\chi^K_{i,t} = \frac{K_{i,t}}{\sum_i K_{i,t}}$ and $\chi^L_{i,t} = \frac{L_{i,t}}{\sum_i L_{i,t}}$ where $K_{i,t}$ is the real capital stock and $L_{i,t}$ is the number of workers in sector i. In comparison, $\gamma_{ij,t}$ and $\chi^y_{i,t}$ are computed

[4]The 2013 version of KLMES reports a capital quantity index for each year. An early vintage of EU-KLEMS (2009 version) contains real capital stock based on 1995 price but the data is only available for a shorter time-series. We construct real capital stock for all years using the 1995 real capital stock and the quantity index.

using expenditure, such that $\gamma_{ij,t} = \frac{\$d_{ij,t}}{\$Q_{j,t}}$ and $\chi^{y}_{j,t} = \frac{\$Y_{j,t}}{\$Q_{j,t}}$ where $\$d_{ij,t}$ is sector i's use of sector j good, $\$Q_{j,t}$ is the nominal value of sector j's gross output and $\$Y_{j,t}$ is the nominal value of sector j good used in final good production.

3.3 Output elasticities in the production functions

Next, to compute the optimal allocation $(\chi^{k*}_{i,t}, \chi^{l*}_{i,t}, \chi^{y*}_{i,t}, \gamma^{*}_{ij,t})$, we need the output elasticities in the production functions ($\alpha_{i,t}$, $\sigma_{ij,t}$, $\lambda_{ij,t}$, $\theta_{i,t}$). Following the literature, we assume that there is no distortion in producing the final good. Therefore θ_i is equal to the expenditure share of sector i good in final goods production. We employ two specifications to back out the rest of the elasticities from the data.

In *specification 1*, we assume that the factor shares in the data are undistorted in each year. Namely, the distortions are in the form of sector-level taxes/subsidies as in Jones (2013), and they are not inputs-specific.

In *specification 2*, we relax this assumption. Similar to Oberfield (2013), we assume that sectors might face input-specific distortions each year, but on average, the factor shares are not distorted. More formally, we take a rolling-window of three years ($t-1, t, t+1$ for the year t) and use the average expenditure share to back out the output elasticities.[5]

We call these specifications year-by-year shares and average shares, respectively.

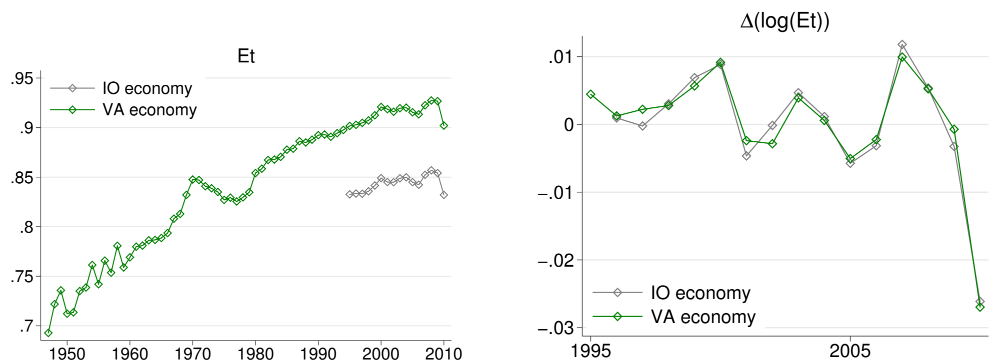
3.4 Slowdown of productivity growth in the US

In this subsection, we study the evolution of allocative efficiency in the US and show how it contributed to aggregate productivity growth.

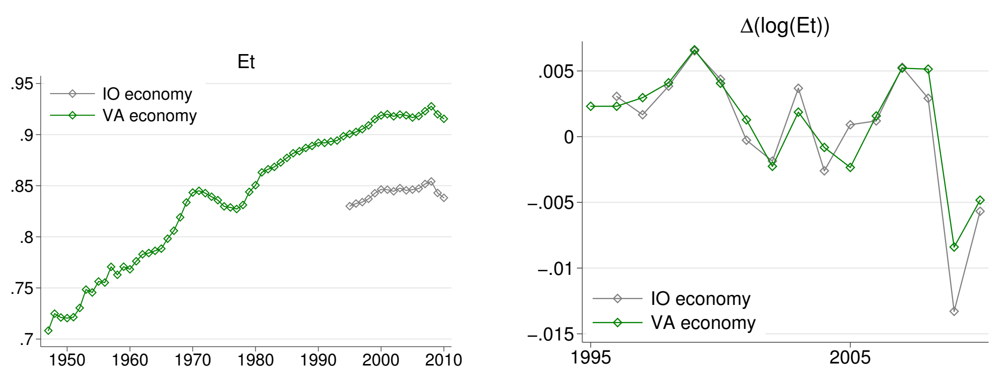
[5] We also try a longer rolling window of 5 years and the result is quantitatively similar. In the long-run, the deviation of expenditure share from the average level in the rolling window could come from both misallocation and technological differences. As a result, we construct the factor shares using a relatively short rolling window to avoid confounding these two factors.

3.4.1 Allocative efficiency over time

Figure 2 displays the allocative efficiency with and without input-output linkages over time. Since solving the allocation problem without input-output linkages does not require information about the input-output structure, the result goes back to 1947. Measured allocative efficiency is higher without input-output linkages, which suggests that missing the linkages leads to underestimating the loss from misallocation (left figures). Despite the difference in level, the two lines show very similar growth rates for the period 1995-2010 when the input-output information is available (right figures). The results are robust across the two specifications in Panel (a) and Panel (b).



(a) specification 1



(b) specification 2

Figure 2: **Allocative efficiency in the US over time**

Source: KLEMS, WIOT, authors' own calculation.
Note: The black line corresponds to the model with the input-output linkage (input-output economy) and the green line is the one without the linkage (value-added economy).

Table 2 shows the contribution of allocative efficiency to the aggregate productivity growth in the US by decade. Except for the 1970s and the 2000s, changes in allocative efficiency contributed significantly to the aggregate productivity growth, ranging from 12 to 32 percent when using the year-by-year shares (specification 1, column 4) and 13 to 34 percent when using the average shares (specification 2, column 6). Over the period 1960-2007, changes in allocative efficiency account for 18 to 20 percent of aggregate productivity growth (last line of column 4 and 6). As a comparison, the contribution of allocative efficiency were at most 1 percent in the 1970s and 3 percent in 2000s (column 6, marked red).

Table 2: **Contribution of allocative efficiency to productivity growth**

	Data	specification 1		specification 2	
	$\Delta log(LP_t)$	$\Delta log\mathbf{E_t}$	$\frac{\Delta log\mathbf{E_t}}{\Delta log(LP_t)}$	$\Delta log\mathbf{E_t}$	$\frac{\Delta log\mathbf{E_t}}{\Delta log(LP_t)}$
(1)	(2)	(3)	(4)	(5)	(6)
1960-1969	+0.24	+0.08	+0.32	+0.08	+0.34
1970-1979	+0.13	-0.02	-0.12	+0.00	+0.01
1980-1989	+0.15	+0.04	+0.27	+0.04	+0.30
1990-1999	+0.19	+0.02	+0.12	+0.03	+0.13
2000-2007	+0.16	+0.00	+0.01	+0.00	+0.03
1960-2007	+0.89	+0.18	+0.20	+0.16	+0.18

Source: BLS, FRED, KLEMS, Author's own calculation.
Note: This table shows the growth rate and the changes in the growth rate of labor productivity for different periods, both in the data and under optimal allocation. dy/y is the growth in labor productivity, measured in log differences, and $\Delta dy/y$ is the change in the growth compared to the previous period. Labor productivity is computed as real output per worker. Allocative efficiency is calculated in the value-added economy.

3.4.2 Slowdown of productivity growth in the 1970s and 2000s

Table 3 displays labor productivity growth in the data and under optimal allocation. As shown in the third column of the Table 3, these two decades (marked red) are characterized by a slowdown in productivity compared with their previous decades. In the data, the growth rate of the 1970s is 12 percentage points lower than that of the 1960s (column 3). Under optimal allocation, however, the slowdown in labor productivity of the 1970s compared with the 1960s is only 3 percentage points (specification 1, column 5) or 4 percentage points (specification 2, column 7). In other words, the slowdown in the improvement in allocation contributes to 2/3 to 3/4 of the slowdown in productivity growth in the 1970s. Similarly, during the 2000s, productivity growth slowed down by 3 percentage points compared to the 1990s (column 3). Under the optimal allocation, however, the growth rate differential between the 2000s and the 1990s is only -1 percentage point (column 5 and 7).

Table 3: **Labor productivity growth in the data and under optimal allocation**

	Data		Optimal specification 1		Optimal specification 2	
	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1960-1969	+0.24		+0.16		+0.16	
1970-1979	+0.13	-0.12	+0.14	-0.03	+0.13	-0.04
1980-1989	+0.15	+0.02	+0.11	-0.02	+0.10	-0.03
1990-1999	+0.19	+0.04	+0.17	+0.06	+0.17	+0.07
2000-2007	+0.16	-0.03	+0.16	-0.01	+0.16	-0.01

Source: BLS, FRED, Author's own calculation

Note: This table shows the growth rate and the changes in the growth rate of labor productivity for different periods, both in the data and under optimal allocation. dy/y is the growth in labor productivity, measured in log differences, and $\Delta dy/y$ is the change in the growth compared to the previous period. Labor productivity is computed as real output per worker.

4 Capital, labor and sectoral allocation

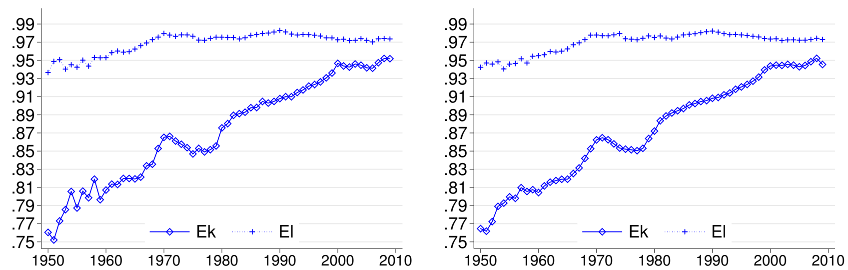
In this section, focusing on the economy without input-output linkage, we explore which factors of production (section 4.1) and sectors (section 4.2) contributed to changes in allocative efficiency.

4.1 Capital and labor

The allocative efficiency can be decomposed into capital and labor allocative efficiency, such that

$$\mathbf{E}_t = E^{k,t} \cdot E^{l,t},$$

where $E^{k,t} = \prod_{i=1}^{N}[(\frac{\chi_{i,t}^{k}}{\chi_{i,t}^{k*}})^{\alpha_{i,t}\theta_{i,t}}$ and $E^{l,t} = \prod_{i=1}^{N}(\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{(1-\alpha_{i,t})\theta_{i,t}}$ are the allocative efficiency of capital and labor, respectively. As shown in Figure 3, capital allocative efficiency improved drastically over the period of 1960 to 1970 and 1980 to 2000 whereas the trend slowed down over the period of 1970 to 1980 and 2000-2007. On the other hand, labor allocative efficiency improved slightly before 1970 and stayed stable from 1970 to 2010. The results imply that the allocation of capital was the main driver behind the overall movement in allocative efficiency both in the long-run and during the productivity slowdown episodes.



(a) specification 1 (b) specification 2

Figure 3: E_t^k **and** E_t^l **over time**

Source: KLEMS, authors' own calculation.

Note: This figure plots evolution of E_k and E_l in a model without input-output linkages.

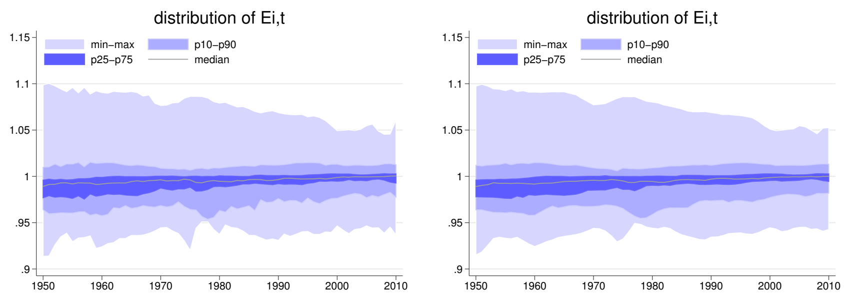
4.2 Sectors

Similarly, the aggregate allocative efficiency can be decomposed into sectoral allocative efficiency $E_{i,t}$, such that,

$$\mathbf{E}_t = \prod_{i=1}^{N} E_{i,t},$$

where $E_{i,t} = [(\frac{\chi_{i,t}^{k}}{\chi_{i,t}^{k*}})^{\alpha_{i,t}}(\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}}]^{\theta_{i,t}}$. If $E_{i,t} < 1$ ($E_{i,t} > 1$) for sector i, it means that capital and labor allocated to this sector is lower (higher) than the optimal level.

Figure 4 plots the distribution of the $E_{i,t}$ over time where different shades of colors represent different percentiles of the $E_{i,t}$ distribution in year t. Under the optimal allocation, sector-level allocative efficiency $E_{i,t} = 1$ for all sectors and the distribution collapses into one point at 1. There was a significant narrowing of the distribution in the US over the period of 1960 to 1970 and 1980 to 2000. In contrast, the distribution became more dispersed in the 1970s and stayed relatively stable after 2000. Intuitively, a more dispersed distribution of $E_{i,t}$ corresponds to a lower aggregate allocative efficiency $\mathbf{E}_t$. The changes in the distribution of E_i in Figure 4 are consistent with the movement of aggregate $\mathbf{E}_t$ in Figure 2.



(a) specification 1 (b) specification 2

Figure 4: **Distribution of $E_{i,t}$ over time**

Source: KLEMS, authors' own calculation.

Note: This figure plots the distribution of $E_{i,t}$ in a model without input-output linkages. The different shades of colors represent different percentiles of the $E_{i,t}$ distribution in year t. The grey line represents the median value of $E_{i,t}$.

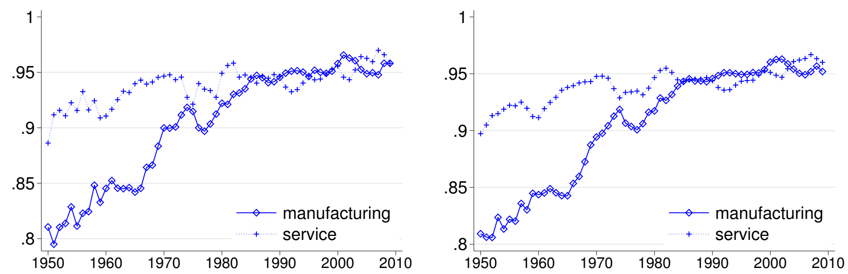
Next, we turn to two broad sets of sectors: the manufacturing and the service sectors.[6] To illustrate changes in allocation of these two sectors, we define E_t^m and E_t^s as the following

$$E_t^m = \prod_{i \in manufacturing} E_{i,t}, \quad E_t^s = \prod_{i \in services} E_{i,t}.$$

Intuitively, E_t^m and E_t^s capture the distance to the optimal allocation of the manufacturing and the service sectors, respectively. As shown in Figure 5, E_t^m and E_t^s increased and moved closer to 1 over the sample period, indicating significant improvement in allocative efficiency for both manufacturing and service sectors. During the 1970s, E_t^m and E_t^s declined

[6]In Table 1, manufacturing sectors include all the sectors under the header "D manufacturing." Service sectors include the sectors under "G Wholesale and retail trade," "I Transportation and storage and communication," "K Real estate, renting and business activities", as well as sector H, J, M, N.

significantly, with the service sectors allocation deteriorated and recovered slightly earlier than the manufacturing sectors. On the other hand, the decline in allocative efficiency was more concentrated in the manufacturing sectors during the 2000s.



(a) specification 1 (b) specification 2

Figure 5: E_t^m **and** E_t^s **over time**

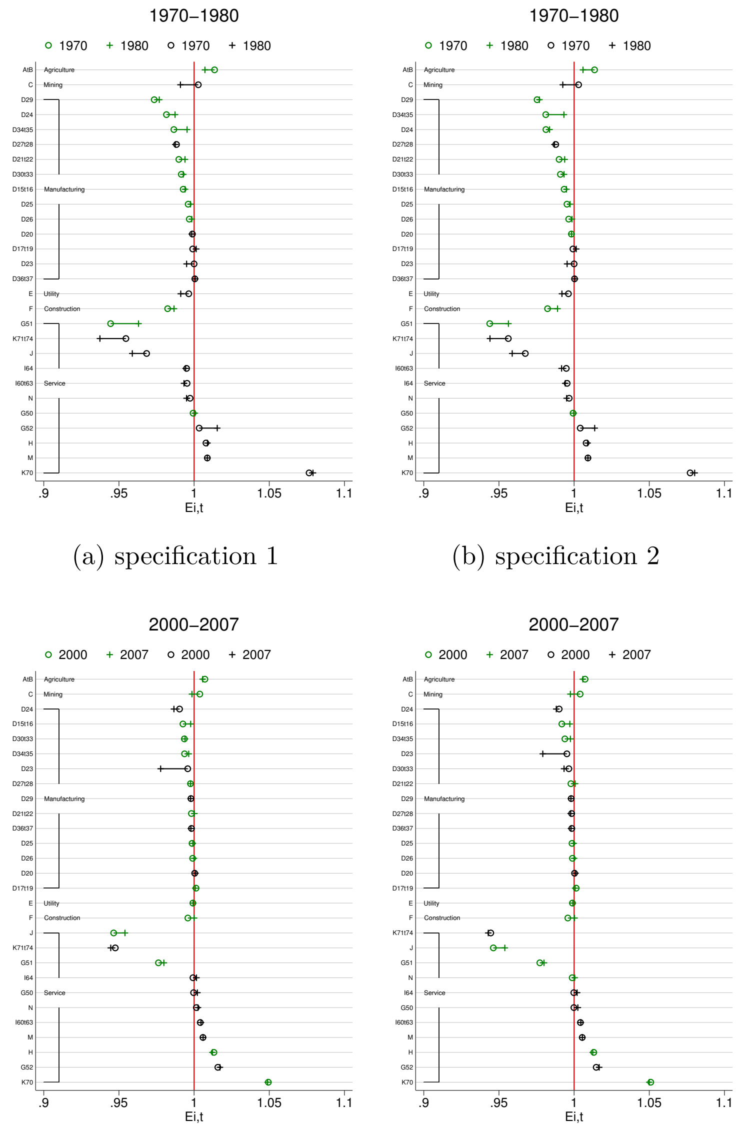
Source: KLEMS, authors' own calculation.

Note: This figure plots E_t^m and E_t^s in a model without input-output linkages.

In measuring E_t^m and E_t^s, the changes in allocative efficiency might be confounded with the shifts in value added shares. For instance, a decline in manufacturing value added share could lead to an increase in E_t^m even if there is no change in allocation. While the shifts in value added shares are small in the short-run, they can be rather significant in the long term, i.e. structural change. To address this issue, we adjust the sectoral weights in E_t^m and E_t^s such that the aggregate value added shares are fixed at 1 for both the manufacturing and service sectors. More formally, we define $\tilde{E}_t^m = (E_t^m)^{1/\sum_{i\in manu}\theta_{i,t}}$ and $\tilde{E}_t^s = (E_t^s)^{1/\sum_{i\in serv}\theta_{i,t}}$. Our results carry over to the re-weighted measures, as shown in Figure A.1.

Finally, we examine the change in $E_{i,t}$ by sector for two periods of 1970-1980 and 2000-2007 in Figure 6. The circle and cross represent the beginning and end of each period, respectively. The distance between the circle and cross then shows the magnitude of the change. We mark the sectors green/black if their allocative efficiency improved/deteriorated during this period ($E_{i,t}$ moved closer to/further away from 1).

Panel (a) and (b) of Figure 6 show that several service sector allocative efficiency deteriorated from 1970 to 1980, most notably for two high-skill sectors of "renting of machine,



(a) specification 1 (b) specification 2

(c) specification 1 (d) specification 2

Figure 6: **Changes in $E_{i,t}$ over 1970-1980 and 2000-2007**

Source: KLEMS, authors' own calculation.

Note: This figure plots the change of $E_{i,t}$ in a model without input-output linkages. The circle and cross represent the beginning and end of each period, respectively. Therefore, the distance between the circle and cross is the magnitude of the change. We mark the sectors green/black if their allocative efficiency improved/deteriorated during this period ($E_{i,t}$ moved closer to/further away from 1).

equipment and other business activities" (K71t74), "financial intermediation" (J) and a low-skill sector of "retail trade" (G52). Importantly, $E_{i,t}$ of the two high-skill service sectors were below 1 in 1970 and declined even further in 1980. This indicates that resources allocated to these sectors moved further below the optimal level. The opposite was true for the retail trade sector: resources allocated to the retail sector moved significantly above the optimal level. From 2000 to 2007, the most notable decline in allocative efficiency happened for the sector that produces "coke refined petroleum products and nuclear fuel" (D23), as shown in Panel (c) and (d), which seems to suggest a connection between the deterioration of allocation and the oil price appreciation in the 2000s.

5 Discussion

In this section, we provide additional discussion about our results. Section 5.1 provides some intuitions on why and how the input-output linkages matter. Section 5.2 studies how the lack of quantity measurement of intermediate inputs affect our results. In section 5.3, we explore how a deviation from the Cobb-Douglas production functions changes measured allocative efficiency.

5.1 Value-added vs. input-output economy

We study both the value-added and the input-output economy with the prior that input-output linkages might affect the measure of allocative efficiency. In this section, we provide some intuition on why and how input-output linkages matter to the measurement. We do this through a simple comparison between the sufficient statistics of allocative efficiency in the value-added and the input-output economy, equation 1 and 4, respectively.

First, not surprisingly, equation 1 does not contain terms that measure the allocation of intermediate inputs. This is simply because capital and labor are the only production factors in the value-added economy.

Second, we turn to the terms that measure capital and labor allocation—equation 1 and the E_{kl} term in equation 4. Both terms are a weighted geometric average of sectoral level allocative efficiency with different sets of sectoral weights. The weights in the value-added economy are θ_i. The weights become $(1-\sigma_i-\lambda_i)\sum_n \theta_n C_{ni}$ in the input-output economy.

Third, the optimal allocation of capital and labor—$\chi^k_{j,t}, \chi^l_{j,t}$—differ as well. Recall that the optimal allocation reflects the relative importance of sectors' capital and labor in producing the final good. In the value-added economy, the relative importance of sector's capital and labor is, again, θ_i whereas in the input-output economy, it is $\frac{\theta_i(1-\sigma_i-\lambda_i)}{1-\sum_j \gamma^*_{ji}}$.

In an undistorted value-added economy, it is clear that θ_i is equal to sector i's value-added share. In fact, it can be shown that in an undistorted input-output economy without international trade, the sectoral weights—$(1-\sigma_i-\lambda_i)\sum_n \theta_n C_{ni}$ and $\frac{\theta_i(1-\sigma_i-\lambda_i)}{1-\sum_j \gamma^*_{ji}}$—are both equal to sector i's value-added shares as well.[7] However, θ_i can potentially differ from $(1-\sigma_i-\lambda_i)\sum_n \theta_n C_{ni}$ and $\frac{\theta_i(1-\sigma_i-\lambda_i)}{1-\sum_j \gamma^*_{ji}}$ in the presence of distortions, which would lead to differences in the measured allocative efficiency between the two economies.

Taking stock, adding input-output linkages alters the measurement of allocative efficiency in two ways: 1) it accounts for the allocation of intermediate inputs, and 2) the set of sectoral weights to measure capital and labor allocation can differ in the presence of distortions.

5.2 Quantity vs. expenditure in measuring allocative efficiency

Due to data limitations, in measuring the cross-sector allocation of intermediate inputs in the data, we use their expenditure rather than quantity (see section 3.2). In this section, we explore what this limitation means for the measurement of allocative efficiency.

We find that the data allocation of intermediate inputs, when measured using expenditure, is always optimal under specification 1. That is, the allocative efficiency of intermediate inputs is 1: $E^d_t = E^m_t = E^y_t = 1$. Therefore, the only type of misallocation that can be identified is that of capital and labor. More formally:

Proposition 4. *Under specification 1 and if the cross-sector allocation of the intermediate inputs in the data ($\gamma_{ij,t}$, $\chi^y_{i,t}$) are computed using expenditure of intermediate inputs, the following equality always hold: $E^d_t = E^m_t = E^y_t = 1$. Therefore, the measured allocative*

[7] See details in section B.5.

efficiency with input-output linkages reduces to

$$\mathbf{E}_t = E_t^{kl} = \prod_{i=1}^{N}(((\frac{\chi_{Ki,t}}{\chi^*_{Ki,t}})^{\alpha_{i,t}}(\frac{\chi_{Li,t}}{\chi^*_{Li,t}})^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t}C_{ni,t}}. \tag{7}$$

Proof. See Appendix B.4. □

The proof of this proposition is rather mechanical, but the proposition has several interesting implications.

First, the findings of this proposition apply to all production factors, including capital and labor. That is, under specification 1, the data allocation of capital and labor, when measured using expenditure (i.e. capital and labor compensation), is also optimal.

Second, recall that allocative efficiency captures the deviation of cross-sector allocation in the data from the optimal allocation. Under specification 1, the cross-sector allocation in the data, when measured using the expenditure of the production factors, is always optimal. Therefore, only when the cross-sector allocation measured using quantity differs from that measured using expenditure can we identify misallocation in the data. In other words, the type of misallocation we can uncover under specification 1 comes from the dispersion of implied prices across sectors.[8] This finding, in a very extreme way, reinforces the understanding in the literature that we need both quantity and expenditure to measure allocative efficiency properly.

Third, under specification 2, the data allocation of intermediate inputs expenditure is not necessarily optimal. However, we find that in the data, the misallocation of intermediate inputs expenditure is quantitatively very small under this specification.

5.3 Elasticity of substitution

So far we operate with the Cobb-Douglas production system. Some research shows that allocative efficiency is affected by the elasticities between production factors (see, e.g. Epifani

[8]We call this "implied" prices because we do not observe price directly in the data. We can infer that there is a dispersion in price across sectors if the allocation of expenditure differs from that of quantity.

and Gancia, 2011 and Osotimehin and Popov, 2020). This section extends the model into one specific type of CES production system and explores how changes in the elasticity could affect our result. More formally, we consider the value added economy as in section 2.1 with a CES aggregator of the final good.

The final good is a CES aggregation of the N intermediate goods, such that

$$Y = (\sum_i \omega_i Y_i^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}}, \tag{8}$$

where ρ measures the elasticity of substitution and ω_i is the weight of good Y_i in the final good production.

The production function of the intermediate good Y_i is the Cobb-Douglas form, such that

$$Y_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i},$$

and the planner solves the following optimization problem

$$max\ Y, s.t \sum_i K_i = K, \sum_i L_i = L.$$

The following proposition characterizes the solution to the problem and the measured allocative efficiency.

Proposition 5. *The allocative efficiency* $\mathbf{E}_t$ *can be written as*

$$\mathbf{E}_t = \frac{Y_t^*}{Y_t} = \{\sum_j \{(\frac{P_j Y_j}{PY})^{\frac{\rho}{\rho-1}} (\frac{\alpha_j / K_j}{\bar{\alpha}^* / K})^{\alpha_j} [\frac{(1-\alpha_j)/L_j}{(1-\bar{\alpha}^*)/L}]^{1-\alpha_j}\}^{\rho-1}\}^{\frac{1}{1-\rho}}, \tag{9}$$

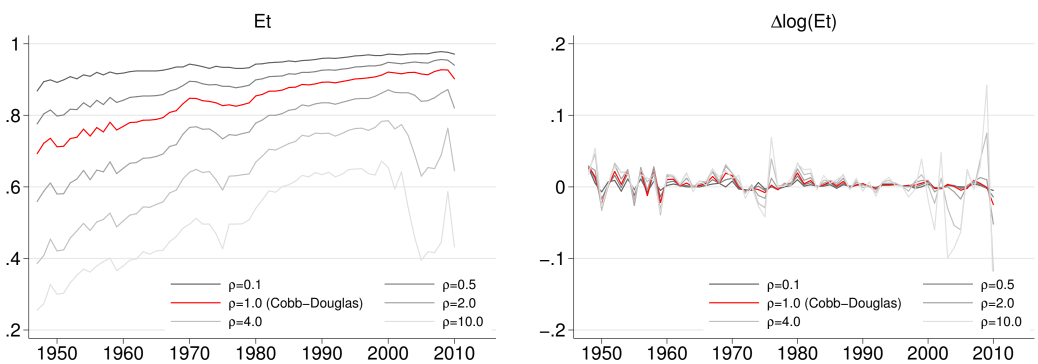
The optimal allocation is characterized by $\{\bar{\alpha}^*, \chi_i^{k*}, \chi_i^{l*}\}$, *such that*

$$\chi_i^{k*} = \frac{K_i^*}{K} = \frac{\frac{P_i Y_i}{PY}\alpha_i}{\bar{\alpha}^*} \quad \chi_i^{l*} = \frac{L_i^*}{L} = \frac{\frac{P_i Y_i}{PY}(1-\alpha_i)}{1-\bar{\alpha}^*}, \quad \bar{\alpha}^* = \sum_i \frac{P_i Y_i}{PY}\alpha_i,$$

where $\frac{P_j Y_j}{PY}$ *is the expenditure share of good* j *in the final good consumption in the data.*

Proof. See Appendix B.6. □

We show the impact of ρ on measured allocative efficiency in Figure 7, where lower ρ means the goods are more complementary to each other. Consistent with the findings in Epifani and Gancia (2011) and Osotimehin and Popov (2020), the level of allocative efficiency is higher when goods are more complementary to each other (Panel a). In addition, the magnitude of changes in $\mathbf{E}_t$, which is more relevant to the decomposition of productivity growth than the level, decreases with complementarity. Panel (a) shows that the growth of $\mathbf{E}_t$ is slower—the line is flatter—with a lower ρ. Panel (b) shows that a lower ρ is also associated with lower volatility in measured $\mathbf{E}_t$. In sum, our benchmark model underestimates (overestimates) the role of allocative efficiency if the goods are more substitutable (complementary) than the Cobb-Douglas case.[9]



(a) (b)

Figure 7: **$\mathbf{E}_t$ and $\Delta log(\mathbf{E}_t)$, different ρ**

Source: KLEMS, authors' own calculation.

Note: This figure plots $\mathbf{E}_t$ and $\Delta log(\mathbf{E}_t)$ without input-output linkages. $\mathbf{E}_t$ is measured under specification 1 and with different value of ρ. Capital is measured using real capitals stock and labor is measured using the number of workers.

[9] On the other hand, however, the elasticities of substitution between inputs are notoriously difficult to measure from the data. The recent development of the literature includes Oberfield and Raval (2020) and Ruane and Peter (2020).

6 Conclusion

In this paper, we ask how much of the slowdown in productivity growth can be explained by factor allocation. We build a tractable framework to measure allocative efficiency and decompose aggregate productivity growth. Applying the theory to the US economy using the KLEMS and WIOT data sets, we show that allocative efficiency can go a long way in explaining productivity slowdown in both 1970s and 2000s. Furthermore, we find that capital allocation was the main driver behind the movements in allocative efficiency, whereas labor allocation stayed relatively unchanged. Both manufacturing and service sectors contributed to productivity slowdown during the 1970s whereas manufacturing sectors played a more significant role in the 2000s.

Recently, the convergence of cross-country per capita income—an old and important question in economic growth—has become topical again (see Johnson and Papageorgiou, 2020 and Startz, 2020). Over the past few decades, countries such as Japan, Korea, and China experienced episodes of fast growth and significantly narrowed the income gap between them and the richest countries in the world. Recent studies show that improvement in allocation was an important contributor behind these "growth miracle" episodes (Buera and Shin, 2013 and Song, Storesletten and Zilibotti, 2011). In general, however, the convergence patterns are rather heterogeneous across countries. In fact, according to Johnson and Papageorgiou (2020), on average, developing countries as a group have not made much progress in converging to the income level of the world frontier. How much of the convergence patterns in the data can be explained by allocative efficiency? The framework developed in this paper can be used to study this question and we leave this for future research.

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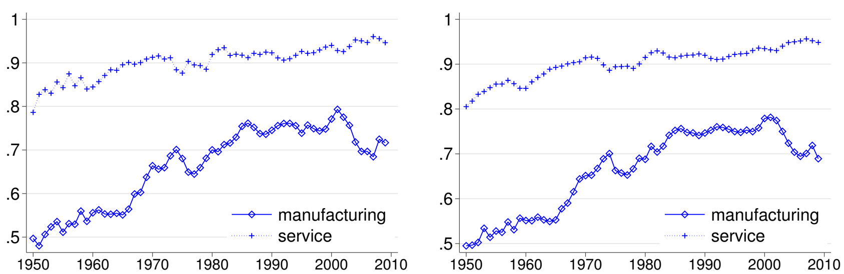
Online Appendix

Not for Publication

A Additional tables and figures



(a) specification 1 (b) specification 2

Figure A.1: $\tilde{E}_t^m$ **and** $\tilde{E}_t^s$ **over time (re-weighted)**

Source: KLEMS, authors' own calculation.
Note: This figure plots $\tilde{E}_t^m$ and $\tilde{E}_t^s$ in a model without input-output linkages where $\tilde{E}_t^m = (E_t^m)^{1/\sum_{i\in manu}\theta_{i,t}}$ and $\tilde{E}_t^s = (E_t^s)^{1/\sum_{i\in serv}\theta_{i,t}}$.

B Proofs and algebra

In the proofs, we drop the time subscript t to simplify notation.

B.1 Proof of proposition 1

The solution to planner's problem requires the equalization of MPK and MPL across sectors, such that,

$$\frac{\partial \log Y}{\partial K_i} = \lambda$$

$$\frac{\partial \log Y}{\partial L_i} = \eta.$$

They can be written as,

$$K_i^* = \frac{\theta_i \alpha_i}{\lambda}$$

$$L_i^* = \frac{\theta_i (1-\alpha_i)}{\eta}$$

Given the resource constraint, we get

$$K_i^* = \chi_{i,t}^{k*} K$$

$$L_i^* = \chi_{i,t}^{l*} L,$$

where $\chi_{i,t}^{k*} = \frac{\theta_i \alpha_i}{\sum_i \theta_i \alpha_i}$ and $\chi_{i,t}^{l*} = \frac{\theta_i (1-\alpha_i)}{\sum_i \theta_i (1-\alpha_i)}$. *Q.E.D.*

B.1.1 Allocative efficiency

The final good output under optimal allocation can be written as

$$\begin{aligned} Y^* &= \prod_i Y_i^{*\theta_i} \\ &= \prod_i (A_i K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{\theta_i} \\ &= \prod_i (A_i (\chi_i^{k*} K)^{\alpha_i} (\chi_i^{l*} L)^{1-\alpha_i})^{\theta_i}. \end{aligned}$$

Similarly the final output in the data is

$$\begin{aligned} Y &= \prod_i Y_i^{\theta_i} \\ &= \prod_i (A_i K_i^{\alpha_i} L_i^{1-\alpha_i})^{\theta_i} \\ &= \prod_i (A_i (\chi_i^k K)^{\alpha_i} (\chi_i^l L)^{1-\alpha_i})^{\theta_i}. \end{aligned}$$

As a result,

$$\mathbf{E}_t = \prod_i \left[\left(\frac{\chi_i^{k*}}{\chi_i^k} \right)^{\alpha_i} \left(\frac{\chi_i^{l*}}{\chi_i^l} \right)^{1-\alpha_i} \right]^{\theta_i}.$$

B.2 Proof of proposition 2

The planner's problem is

$$C = \prod_{i=1}^{N} (Q_i - \sum_{j=1}^{N} d_{ji})^{\theta_i} - \sum_i \sum_j \bar{P}_j m_{ij}.$$

The FOCs for K_i, L_i, d_{ij}, m_{ij} are

$$\begin{aligned} \frac{\partial C}{\partial K_i} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{K_i^*} \alpha_i (1 - \sigma_i - \lambda_i) \\ \frac{\partial C}{\partial L_i} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{K_i^*} (1 - \alpha_i)(1 - \sigma_i - \lambda_i) \\ \frac{\partial C}{\partial d_{ij}} &= \theta_i \frac{Y^*}{Y_i^*} [\frac{Q_i^*}{d_{ij}^*} \sigma_{ij} - I_{\{i=j\}}] + \theta_j \frac{Y^*}{Y_j^*} [\frac{Q_j^*}{d_{jj}^*} \sigma_{jj} I_{\{i=j\}} - 1] \\ \frac{\partial C}{\partial m_{ij}} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{m_{ij}^*} \lambda_{ij} - \bar{P}_j \end{aligned}$$

The FOC $\frac{\partial C}{\partial d_{ij}} = 0$ implies

$$d_{ij}^* = \frac{\theta_i Y_j^*}{\theta_j Y_i^*} \sigma_{ij} Q_i^*, \tag{10}$$

therefore

$$Y_j^* = Q_j^* - \sum_{i=1}^N d_{ij}^* = Q_j^* - \sum_{i=1}^N \frac{\theta_i Y_j^*}{\theta_j Y_i^*} \sigma_{ij} Q_i^*,$$
$$Y_j^*[1 + \frac{1}{\theta_j} \sum_i (\frac{\theta_i Q_i^*}{Y_i^*} \sigma_{ij})] = Q_j^*.$$

Let $\chi_j^{y*} = \frac{Y_i^*}{Q_i^*}$, $\{\chi_i^{y*}\}_{i=1}^N$ solve the following equations

$$\frac{1}{\chi_i^{y*}} = 1 + \frac{1}{\theta_i} \sum_s (\frac{\theta_s}{\chi_s^{*y}} \sigma_{si}) \tag{11}$$

or

$$1 - \chi_j^{y*} = \sum_i \sigma_{ij} \frac{\theta_i \chi_j^{y*}}{\theta_j \chi_i^{y*}}.$$

Let $\gamma_{ij}^* = \frac{\theta_i \chi_j^{y*}}{\theta_j \chi_i^{y*}} \sigma_{ij}$ in equation 10, then $d_{ij}^* = \gamma_{ij}^* Q_j^*$. The market clear condition for Q_i^* implies

$$\chi_i^{y*} = 1 - \sum_s \gamma_{si}^*.$$

FOC $\frac{\partial C}{\partial m_{ij}} = 0$ implies

$$m_{ij}^* = \theta_i \frac{Y^*}{Y_i^*} Q_i^* \frac{\lambda_{ij}}{\bar{P}_j} \tag{12}$$

Since

$$Y^* = \prod_i Y_i^{*\theta_i} = \prod_i (\chi_i^* Q_i^*)^{\theta_i}$$

we have

$$m_{ij}^* = \theta_i \prod_s (\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s} \prod_s (Q_s^*)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j} \tag{13}$$

The FOC $\frac{\partial C}{\partial K_i} = 0$ and $\frac{\partial C}{\partial L_i} = 0$ lead to

$$K_i^* = \chi_i^{k*} K \tag{14}$$

$$L_i^* = \chi_i^{l*} L \tag{15}$$

where

$$\chi_i^{k*} = \frac{\frac{\theta_i\alpha_i(1-\sigma_i-\lambda_i)}{(1-\sum_j \gamma_{ji}^*)}}{\sum_s \frac{\theta_s\alpha_s(1-\sigma_s-\lambda_s)}{(1-\sum_j \gamma_{js}^*)}}, \chi_i^{l*} = \frac{\frac{\theta_i(1-\alpha_i)(1-\sigma_i-\lambda_i)}{(1-\sum_j \gamma_{ji}^*)}}{\sum_s \frac{\theta_s(1-\alpha_s)(1-\sigma_s-\lambda_s)}{(1-\sum_j \gamma_{js}^*)}}. \tag{16}$$

To fully characterize d_{ij} and m_{ij},we need to solve for Q_i. Replace d_{ij} and m_{ij} in the production function using $d_{ij}^* = \gamma_{ij}^* Q_j^*$ and 13, we get

$$\begin{aligned} Q_i^* &= A_i(K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\gamma_{i1}^* Q_1^*)^{\sigma_{i1}} \cdots (\gamma_{iN}^* Q_N^*)^{\sigma_{iN}} \prod_{j=1}^N \{\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s} \prod_s (Q_s^*)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j}\}^{\lambda_{ij}} \quad (17) \\ &= A_i(K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})(\prod_{j=1}^N Q_j^{*\sigma_{ij}})[\prod_s (Q_s^*)^{\theta_s}]^{\lambda_i}[\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}} \\ &= A_i[(\chi_i^{k*}K)^{\alpha_i}(\chi_i^{l*}L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}} (\prod_{s=1}^N Q_s^{*\sigma_{is}+\lambda_i\theta_s}). \end{aligned}$$

Define

$$\chi_{Qi}^* = A_i[(\chi_i^{k*}K)^{\alpha_i}(\chi_i^{l*}L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}} \tag{18}$$

The above equation can be written as

$$Q_i^* = \chi_{Qi}^* (\prod_{s=1}^N Q_s^{*\sigma_{is}+\lambda_i\theta_s}). \tag{19}$$

Q.E.D.

B.2.1 Allocative efficiency

Taking log of equation 19 gives $\log Q_i^* = \log\chi_{Qi}^* + \sum_{j=1}^N[(\sigma_{ij} + \lambda_i\theta_j)\log(Q_j^*)]$. Let $q^* = [\log(Q_1^*), \ldots, \log(Q_N^*)]'_{N\times 1}$, equation 19 can be written as

$$q_{N\times 1}^* = b_{N\times 1}^* + \Omega_{N\times N} q_{N\times 1}^*,$$

where $b^*(i) = \log\chi_{Qi}^*$ and $\Omega(i,j) = \sigma_{ij} + \lambda_i\theta_j$. Therefore q can be solved as $q = Cb^*$ where $C_{N\times N} = (I - \Omega)^{-1}$ and $Q_n^* = \prod_{i=1}^N(\chi_{Qi}^{*C_{ni}})$.

Rewrite equation 18 as

$$\chi_{Qi}^{*} = z_i^{*} K^{\alpha_i(1-\sigma_i-\lambda_i)} L^{(1-\alpha_i)(1-\sigma_i-\lambda_i)}$$

where $z_i^{*} = A_i[(\chi_i^{k*})^{\alpha_i}(\chi_i^{l*})^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$.

Then Q_n^{*} can be rewritten as

$$Q_n^{*} = \prod_{i=1}^{N}(\chi_{Qi}^{*C_{ni}}) = \tilde{A}_n^{*} K^{\tilde{\alpha}_n} L^{\tilde{\beta}_n} \tag{20}$$

where $\tilde{A}_n^{*} = \{\prod_{i=1}^{N} z_i^{*C_{ni}}\}$, and $\tilde{\alpha}_n = \sum_i(\alpha_i(1-\sigma_i-\lambda_i)C_{ni})$, $\tilde{\beta}_n = \sum_i((1-\alpha_i)(1-\sigma_i-\lambda_i)C_{ni})$.

Aggregate output under optimal allocation can be written as a function of aggregate capital K and aggregate labor L

$$Y^{*} = \prod_i Y_i^{*\theta_i} = \prod_i(\chi_i^{y*}\tilde{A}_i^{*}K^{\tilde{\alpha}_i}L^{\tilde{\beta}_i})^{\theta_i} = \bar{A}^{*}K^{\bar{\alpha}}L^{\bar{\beta}}, \tag{21}$$

where $\bar{A}^{*} = \prod_{i=1}^{N}(\chi_i\tilde{A}_i^{*})^{\theta_i}$ is the aggregate TFP under optimal allocation, and $\bar{\alpha} = \sum_n(\tilde{\alpha}_n\theta_n)$, $\bar{\beta} = \sum_n(\tilde{\beta}_n\theta_n)$.

Replacing Q_s^{*} in equation 13 using the expression in 20, we can write the expenditure on imported good j as

$$\bar{P}_j m_{ij}^{*} = [\prod_s(\chi_s^{y*}\tilde{A}_s^{*})^{\theta_s}]\{\frac{\theta_i}{\chi_i^{y*}}K^{\sum_s\theta_s\tilde{\alpha}_s}L^{\sum_s\theta_s\tilde{\beta}_s}\}\lambda_{ij} = (\frac{\theta_i\lambda_{ij}}{\chi_i^{y*}})Y^{*}.$$

The total expenditure on imported goods is

$$X^{*} = [\sum_{i=1}^{N}(\frac{\theta_i\lambda_i}{\chi_i^{y*}})]Y^{*}.$$

The output net of imported goods is

$$C^{*} = Y^{*} - X^{*} = Y^{*}[1 - \sum_{i=1}^{N}(\frac{\theta_i\lambda_i}{\chi_i^{y*}})].$$

Next, we write the data output Y as a function of data allocation (without the stars). The data analog of equation 17 is

$$Q_i = A_i(K_i^{\alpha_i} L_i^{1-\alpha_i})^{1-\sigma_i-\lambda_i}(\gamma_{i1}Q_1)^{\sigma_{i1}} \cdots (\gamma_{iN}Q_N)^{\sigma_{iN}} \prod_{j=1}^{N}\{\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s} \prod_s (Q_s)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j}\}^{\lambda_{ij}}.$$

Let $\chi_{Qi} = A_i[(\chi_i^k K)^{\alpha_i}(\chi_i^l L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N} \gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$. The above equation can be written as

$$Q_i = \chi_{Qi}(\prod_{s=1}^{N} Q_s^{\sigma_{is}+\lambda_i\theta_s}).$$

Let $q = [\log(Q_1), \ldots, \log(Q_N)]'_{N\times 1}$, we can solve q as

$$q_{N\times 1} = b_{N\times 1} + \Omega_{N\times N} q_{N\times 1},$$

where $b(i) = \log\chi_{Qi}$ and $\Omega(i,j) = \sigma_{ij} + \lambda_i\theta_j$. Therefore q can be solved as $q = Cb$ where $C_{N\times N} = (I - \Omega)^{-1}$. Therefore,

$$Q_n = \prod_{i=1}^{N}(\chi_{Qi}^{C_{ni}}) = \tilde{A}_n K^{\tilde{\alpha}_n} L^{\tilde{\beta}_n}.$$

where $\tilde{A}_n = \{\prod_{i=1}^{N} z_i{}^{C_{ni}}\}$, and $\tilde{\alpha}_n = \sum_i(\alpha_i(1-\sigma_i-\lambda_i)C_{ni})$, $\tilde{\beta}_n = \sum_i((1-\alpha_i)(1-\sigma_i-\lambda_i)C_{ni})$. We can write the data output as

$$Y = \prod_i Y_i^{\theta_i} = \prod_i (\chi_i^y \tilde{A}_i K^{\tilde{\alpha}_i} L^{\tilde{\beta}_i})^{\theta_i} = \bar{A} K^{\bar{\alpha}} L^{\bar{\beta}}, \tag{22}$$

where $\bar{A} = \prod_{i=1}^{N}(\chi_i^y \tilde{A}_i)^{\theta_i}$ is the aggregate TFP in the data.

In addition, we assume that the expenditure shares of imported intermediate goods are not distorted, such that

$$\bar{P}_j m_{ij} = \lambda_{ij} P_i Q_i = \frac{\lambda_{ij} P_i Y_i}{\chi_i^y} = \frac{\theta_i \lambda_{ij}}{\chi_i^y} Y.$$

Thus

$$X = [\sum_{i=1}^{N}(\frac{\theta_i \lambda_i}{\chi_i^y})]Y$$

and

$$C = Y - X = (1 - \sum_{i=1}^{N}(\frac{\theta_i \lambda_i}{\chi_i^y}))Y.$$

Now we can compute the allocative efficiency as

$$\mathbf{E} = \frac{C}{C^*} = \frac{[1 - \sum_{n=1}^{N}(\frac{\theta_n \lambda_n}{\chi_n^y})] \prod_{n=1}^{N}(\chi_n^y \tilde{A}_n)^{\theta_n} K^{\bar{\alpha}} L^{\bar{\beta}}}{[1 - \sum_{n=1}^{N}(\frac{\theta_n \lambda_n}{\chi_n^{y*}})] \prod_{n=1}^{N}(\chi_n^{y*} \tilde{A}_n^*)^{\theta_n} K^{\bar{\alpha}} L^{\bar{\beta}}},$$

where

$$\begin{aligned}\frac{\tilde{A}_n}{\tilde{A}_n^*} &= \prod_{i=1}^{N}\{\frac{A_i(\chi_i^{k\alpha_i}\chi_i^{l1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}}{A_i(\chi_i^{k*\alpha_i}\chi_i^{l*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{*\sigma_{ij}})[\theta_i \prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}}\}^{C_{ni}} \\ &= \prod_{i=1}^{N}\{[(\frac{\chi_i^k}{\chi_i^{k*}})^{\alpha_i}(\frac{\chi_i^l}{\chi_i^{l*}})^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}\frac{[\prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}]^{\lambda_i}}{[\prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}]^{\lambda_i}}\prod_{j=1}^{N}(\frac{\gamma_{ij}}{\gamma_{ij}^*})^{\sigma_{ij}}\}^{C_{ni}}.\end{aligned}$$

Rearrange, we get,

$$\mathbf{E} = E^{kl}E^{d}E^{m}E^{y},$$

where $E^{kl} = \prod_{i=1}^{N}((((\frac{\chi_i^k}{\chi_i^{k*}})^{\alpha_{i,t}}(\frac{\chi_i^l}{\chi_i^{l*}})^{1-\alpha_i})^{1-\sigma_i-\lambda_i})^{\sum_n \theta_n C_{ni}}$, $E^d = \prod_{i=1}^{N}(\prod_{j=1}^{N}(\frac{\gamma_{ij}}{\gamma_{ij}^*})^{\sigma_{ij}})^{\sum_n \theta_n C_{ni}}$, $E^m = \frac{1-\sum_{n=1}^{N}\frac{\theta_n \lambda_n}{\chi_n^y}}{1-\sum_{n=1}^{N}\frac{\theta_n \lambda_n}{\chi_n^{y*}}}$ and $E^y = \prod_{n=1}^{N}(\frac{\chi_n^y}{\chi_n^{y*}})^{\theta_{n,t}}\prod_{i=1}^{N}(\frac{\prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}}{\prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}})^{\lambda_i \sum_n(\theta_n C_{ni})}$.

In addition, we can show that the value-added aggregate production function that features a constant returns to scale. That is, $\bar{\alpha} + \bar{\beta} = 1$. To show this, we only need to show that $(\tilde{\alpha}_n + \tilde{\beta}_n) = 1$. Then it follows that $\bar{\alpha} + \bar{\beta} = \sum_n((\tilde{\alpha}_n + \tilde{\beta}_n)\theta_n) = \sum_n \theta_n = 1$.

To show that $\tilde{\alpha}_n + \tilde{\beta}_n = 1$, let $B = I - \Omega$, therefore

$$\sum_j B(i,j) = \sum_j(1 - (\sigma_i + \lambda_i\theta_j)) = 1 - (\sigma_i + \lambda_i).$$

The first equality is because of the definition of Ω. The second equality holds because

$\sum_j \theta_j = 1$. Note that

$$\tilde{\alpha}_n + \tilde{\beta}_n = \sum_i (C_{ni}(1 - \sigma_i - \lambda_i)) = \sum_i \sum_j C(n,i)B(i,j)$$

Since by definition, $BC = CB = I$, $\sum_j \sum_i C(n,i)B(i,j) = 1$ holds for any n. Therefore $(\tilde{\alpha}_n + \tilde{\beta}_n) = 1$.

B.3 Proof of Proposition 3

According to the definition of E_t, the following equation holds: $Y_t = Y_t^* \mathbf{E_t}$ or $C_t = C_t^* \mathbf{E_t}$. Dividing both sides by the aggregate labor inputs yields $LP_t = LP_t^* \mathbf{E_t}$. Taking the log difference on both side yields $\Delta log(LP_t) = \Delta log(LP_t^*) + \Delta log \mathbf{E_t}$. *Q.E.D.*

B.4 Proof of Proposition 4

To prove this proposition, we need to show that $E_t^m = E_t^d = E_t^y = 1$ in equation 4. It is sufficient to show that $\chi_{i,t}^y = \chi_{i,t}^{y*}$, where $\chi_{i,t}^{y*}$ is the optimal allocation and $\chi_{i,t}^y$ is its data analog.

To simplify notations, we drop the time subscript. Note that x_i^{y*} is a solution to the system of equations 2. We intend to show that χ_i^y also satisfy 2, which we reproduced here: $\frac{1}{\chi_i^y} = 1 + \frac{1}{\theta_i} \sum_s (\frac{\theta_s}{\chi_s^y} \sigma_{si})$. Let $\eta_i = \frac{1}{\chi_i^y}$, the above system of equations can be written as,

$$\eta_i = 1 + \sum_s (\eta_s \frac{\theta_s}{\theta_i} \sigma_{si}), \forall i \in \{1, ..., N\},$$

equivalently,

$$\begin{pmatrix} \eta_1 - 1 \\ \vdots \\ \eta_N - 1 \end{pmatrix} = \underbrace{\begin{pmatrix} \sigma_{11} & \cdots & \frac{\theta_N}{\theta_1} \sigma_{N1} \\ \vdots & \ddots & \vdots \\ \frac{\theta_1}{\theta_N} \sigma_{1N} & \cdots & \sigma_{NN} \end{pmatrix}}_{\Pi} \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_N \end{pmatrix} \tag{23}$$

in which $\Pi(i,j) = \frac{\theta_j}{\theta_i} \sigma_{ji}$.

We compute the data allocation of intermediate inputs as

$$\eta_i = \frac{1}{\chi_i^y} = \frac{\$Q_i}{\$Y_i},$$

where the dollar sign \$ indicates a measure of expenditure (nominal value).

Under specification 1, we assume that the expenditure shares are undistorted in each year and thus they are equal to the output elasticities in the production functions. More specifically, under this specification, the elasticities θ_i and σ_{ij} are calculated as

$$\sigma_{ij}\theta_i = \frac{\$Y_i}{\sum_s \$Y_s}, \quad \sigma_{ij} = \frac{\$d_{ij}}{\$Q_i}.$$

Take the data η_i, σ_{ij} and θ_i back to the equation 23, the RHS can be written as

$$\begin{pmatrix} \frac{\$d_{11}}{\$Q_1} & \cdots & \frac{\$Y_N}{\$Q_N}\frac{\$d_{N1}}{\$Y_1} \\ \vdots & \ddots & \vdots \\ \frac{\$Y_1}{\$Q_1}\frac{\$d_{1N}}{\$Y_N} & \cdots & \frac{\$d_{NN}}{\$Q_N} \end{pmatrix} \begin{pmatrix} \frac{\$Q_1}{\$Y_1} \\ \vdots \\ \frac{\$Q_N}{\$Y_N} \end{pmatrix} = \begin{pmatrix} \frac{\sum_{s=1}^N \$d_{s1}}{\$Y_1} \\ \vdots \\ \frac{\sum_{s=1}^N \$d_{sN}}{\$Y_N} \end{pmatrix} = \begin{pmatrix} \frac{\$Q_1 - \chi_1 \$Q_1}{\chi_1 \$Q_1} \\ \vdots \\ \frac{\$Q_N - \chi_N \$Q_N}{\chi_N \$Q_N} \end{pmatrix} = \begin{pmatrix} \eta_1 - 1 \\ \vdots \\ \eta_N - 1 \end{pmatrix},$$

which is equal to the LHS of the equation. The first equality is simple algebra, the second equality holds because of the market clear condition for Q_i and the third equality is because of the definition of η_i.[10] *Q.E.D.*

B.5 Comparing sectoral weights in undistorted economies

In section 5.1, we compare the sufficient statistics for measuring allocative efficiency in the value-added and the input-output economy. We reproduce the two sets of weights in the table below. Next we show that, in an undistorted economy without international trade, the weights listed in the table are all equal to sectoral value-added shares.

- θ_i in the value-added economy. The FOC of the final good producer gives $\theta_i = \frac{P_i Y_i}{Y}$,

[10]Note that if intermediate inputs are measured using their quantity, i.e., $\eta_i = \frac{Q_i}{Y_i}$, the first equality no longer holds.

Table B.1: Sectoral weights in value-added versus input-output economies

	value-added	input-output
1st set	θ_i	$(1-\sigma_i-\lambda_i)\sum_n \theta_n C_{nj}$
2nd set	θ_i	$\frac{\theta_i(1-\sigma_i-\lambda_i)}{1-\sum_j \gamma^*_{ji}}$

where $P_i Y_i$ is the value-added output of sector i and Y is the total value-added output (final good price normalized to 1).

- $(1-\sigma_i-\lambda_i)\sum_n \theta_n C_{nj}$ in the input-output economy. Note that without international trade, matrix C is just the Leontief inverse matrix. Therefore $\sum_n \theta_n C_{ni}$ is equal to the undistorted Domar weight—sector i's sales (gross output) to GDP. Multiplying it by sector i's value-added share $(1-\sigma_i-\lambda_i)$ yields the value-added share of sector i.

- $\frac{\theta_i(1-\sigma_i-\lambda_i)}{1-\sum_j \gamma^*_{ji}}$ in the input-output economy. We only need to show that $\frac{\theta_i}{1-\sum_j \gamma^*_{ji}}$ is equal to the undistorted Domar weight. Note that the star here indicates that this is the share under optimal allocation: $\frac{\theta_i}{1-\sum_j \gamma^*_{ji}} = \frac{\theta_i}{\chi^*_i} = \frac{P_i \theta_i Q^*_i}{P_i Y^*_i} = \frac{P_i Q^*_i}{Y^*}$. The second equality holds because of the definition of χ^*_i, such that $\chi^*_i = Y^*_i/Q^*_i$. The last equality holds because of the FOC of the final good producer $\theta_i Y^* = P_i Y^*_i$.

B.6 Proof of Proposition 5

The FOCs of the planner's problem give

$$\begin{aligned} \omega_i(\frac{Y_i}{Y})^{1-\frac{1}{\rho}} &= \frac{K_i/\alpha_i}{\sum_i (K_i/\alpha_i)}, \\ \omega_i(\frac{Y_i}{Y})^{1-\frac{1}{\rho}} &= \frac{L_i/(1-\alpha_i)}{\sum_i [L_i/(1-\alpha_i)]}. \end{aligned}$$

To simplify notations, we denote $\tilde{K}_i = K_i/\alpha_i$, $\tilde{L}_i = L_i/(1-\alpha_i)$ and define $\tilde{K} = \sum_i \tilde{K}_i$ and $\tilde{L} = \sum_i \tilde{L}_i$. It is clear that, from the FOCs, $\frac{\tilde{K}_i}{\tilde{L}_i} = \frac{\sum_i \tilde{K}_i}{\sum_i \tilde{L}_i} = \frac{\tilde{K}}{\tilde{L}}$. We can rewrite K_i and L_i

using the production functions as

$$
\begin{aligned}
K_i &= (\alpha_i \tilde{K})\omega_i^{\rho}[\frac{A_i(\alpha_i \tilde{K})^{\alpha_i}[(1-\alpha_i)\tilde{L}]^{1-\alpha_i}}{Y}]^{\rho-1}, \\
L_i &= (1-\alpha_i)\tilde{L}\omega_i^{\rho}[\frac{A_i(\alpha_i \tilde{K})^{\alpha_i}[(1-\alpha_i)\tilde{L}]^{1-\alpha_i}}{Y}]^{\rho-1}.
\end{aligned}
$$

Given $\sum_i K_i = K$,$\sum_i L_i = L$, we can solve $Y, \tilde{K}, \tilde{L}$ with the system of three equations

$$
\begin{aligned}
K &= \sum_i \{(\alpha_i \tilde{K})\omega_i^{\rho}[\frac{A_i(\alpha_i \tilde{K})^{\alpha_i}[(1-\alpha_i)\tilde{L}]^{1-\alpha_i}}{Y}]^{\rho-1}\}, \\
L &= \sum_i \{(1-\alpha_i)\tilde{L}\omega_i^{\rho}[\frac{A_i(\alpha_i \tilde{K})^{\alpha_i}[(1-\alpha_i)\tilde{L}]^{1-\alpha_i}}{Y}]^{\rho-1}\}, \\
Y^{\rho-1} &= \sum_i \omega_i^{\rho}\{A_i(\alpha_i \tilde{K})^{\alpha_i}[(1-\alpha_i)\tilde{L}]^{1-\alpha_i}\}^{\rho-1}.
\end{aligned}
$$

In particular,

$$
\begin{aligned}
\frac{K}{\tilde{K}} &= \sum_i \{\alpha_i \frac{\omega_i^{\rho}\{A_i(\alpha_i \tilde{K})^{\alpha_i}[(1-\alpha_i)\tilde{L}]^{1-\alpha_i}\}^{\rho-1}}{\sum_j \omega_j^{\rho}\{A_j(\alpha_j \tilde{K})^{\alpha_j}[(1-\alpha_j)\tilde{L}]^{1-\alpha_j}\}^{\rho-1}}\}, \\
\frac{L}{\tilde{L}} &= \sum_i \{(1-\alpha_i)\frac{\omega_i^{\rho}\{A_i(\alpha_i \tilde{K})^{\alpha_i}[(1-\alpha_i)\tilde{L}]^{1-\alpha_i}\}^{\rho-1}}{\sum_j \omega_j^{\rho}\{A_j(\alpha_j \tilde{K})^{\alpha_j}[(1-\alpha_j)\tilde{L}]^{1-\alpha_j}\}^{\rho-1}}\},
\end{aligned}
$$

and $\frac{K}{\tilde{K}} + \frac{L}{\tilde{L}} = 1$.

Denote $\bar{\alpha} = \frac{K}{\tilde{K}}$, then $\frac{L}{\tilde{L}} = 1 - \bar{\alpha}$, and $\bar{\alpha}$ solves the following equation

$$
\bar{\alpha} = \sum_i \{\alpha_i \frac{\omega_i^{\rho}\{A_i(\frac{\alpha_i}{\bar{\alpha}}K)^{\alpha_i}[\frac{(1-\alpha_i)}{(1-\bar{\alpha})}L]^{1-\alpha_i}\}^{\rho-1}}{\sum_j \omega_j^{\rho}\{A_j(\frac{\alpha_j}{\bar{\alpha}}K)^{\alpha_j}[\frac{(1-\alpha_j)}{(1-\bar{\alpha})}L]^{1-\alpha_j}\}^{\rho-1}}\}, \tag{24}
$$

and the output under optimal allocation is

$$
Y^* = \{\sum_j \omega_j^{\rho}\{A_j(\frac{\alpha_j}{\bar{\alpha}}K)^{\alpha_j}[\frac{(1-\alpha_j)}{(1-\bar{\alpha})}L]^{1-\alpha_j}\}^{\rho-1}\}^{\frac{1}{\rho-1}}.
$$

If we replace, in the previous equation, A_i with $\frac{Y_i}{K_i^{\alpha_i}L_i^{1-\alpha_i}}$ and Y_i with $(\frac{P_i Y_i/\omega_j}{PY})^{\frac{\rho}{\rho-1}}Y$, we can rewrite the allocative efficiency as

$$\mathbf{E} = \frac{Y}{Y^*} = \{\sum_j \omega_j^\rho \{ \frac{(\frac{P_jY_j/\omega_j}{PY})^{\frac{\rho}{\rho-1}}}{K_j^{\alpha_j} L_j^{1-\alpha_j}} (\frac{\alpha_j}{\bar{\alpha}} K)^{\alpha_j} [\frac{(1-\alpha_j)}{(1-\bar{\alpha})} L]^{1-\alpha_j} \}^{\rho-1} \}^{\frac{1}{1-\rho}},$$
$$= \{\sum_j \{ (\frac{P_jY_j}{PY})^{\frac{\rho}{\rho-1}} (\frac{\alpha_j/K_j}{\bar{\alpha}/K})^{\alpha_j} [\frac{(1-\alpha_j)/L_j}{(1-\bar{\alpha})/L}]^{1-\alpha_j} \}^{\rho-1} \}^{\frac{1}{1-\rho}},$$

which means that E is a function of $\bar{\alpha}$, the expenditure share $\frac{P_jY_j}{PY}$ in the data, capital allocation $\frac{K_j}{K}$ and labor allocation $\frac{L_j}{L}$ in the data. All the other measures, except for $\bar{\alpha}$ are clearly unit-less. We show next that so is $\bar{\alpha}$. By replacing A_i in equation 24, we can write the equation as the following,

$$\bar{\alpha} = \sum_i \{ \alpha_i \frac{\omega_i^\rho \{ A_i (\frac{\alpha_i}{\bar{\alpha}} K)^{\alpha_i} [\frac{(1-\alpha_i)}{(1-\bar{\alpha})} L]^{1-\alpha_i} \}^{\rho-1}}{\sum_j \omega_j^\rho \{ A_j (\frac{\alpha_j}{\bar{\alpha}} K)^{\alpha_j} [\frac{(1-\alpha_j)}{(1-\bar{\alpha})} L]^{1-\alpha_j} \}^{\rho-1}} \}$$
$$= \sum_i \{ \alpha_i \frac{\omega_i^\rho \{ \frac{Y_i}{K_i^{\alpha_i} L_i^{1-\alpha_i}} (\frac{\alpha_i}{\bar{\alpha}} K)^{\alpha_i} [\frac{(1-\alpha_i)}{(1-\bar{\alpha})} L]^{1-\alpha_i} \}^{\rho-1}}{\sum_j \omega_j^\rho \{ \frac{Y_j}{K_j^{\alpha_j} L_j^{1-\alpha_j}} (\frac{\alpha_j}{\bar{\alpha}} K)^{\alpha_j} [\frac{(1-\alpha_j)}{(1-\bar{\alpha})} L]^{1-\alpha_j} \}^{\rho-1}} \}$$
$$= \sum_i \{ \alpha_i \frac{\omega_i^\rho \{ (\frac{P_iY_i/\omega_j}{PY})^{\frac{\rho}{\rho-1}} Y (\frac{\alpha_i}{\bar{\alpha}} K/K_i)^{\alpha_i} [\frac{(1-\alpha_i)}{(1-\bar{\alpha})} L/L_i]^{1-\alpha_i} \}^{\rho-1}}{\sum_j \omega_j^\rho \{ (\frac{P_jY_j/\omega_j}{PY})^{\frac{\rho}{\rho-1}} Y (\frac{\alpha_j}{\bar{\alpha}} K/K_j)^{\alpha_j} [\frac{(1-\alpha_j)}{(1-\bar{\alpha})} L/L_j]^{1-\alpha_j} \}^{\rho-1}} \}$$
$$= \sum_i \{ \alpha_i \frac{Y^{\rho-1} \{ (\frac{P_iY_i}{PY})^{\frac{\rho}{\rho-1}} (\frac{\alpha_i}{\bar{\alpha}} K/K_i)^{\alpha_i} [\frac{(1-\alpha_i)}{(1-\bar{\alpha})} L/L_i]^{1-\alpha_i} \}^{\rho-1}}{\sum_j Y^{\rho-1} \{ (\frac{P_jY_j/\omega_j}{PY})^{\frac{\rho}{\rho-1}} (\frac{\alpha_j}{\bar{\alpha}} K/K_j)^{\alpha_j} [\frac{(1-\alpha_j)}{(1-\bar{\alpha})} L/L_j]^{1-\alpha_j} \}^{\rho-1}} \}$$
$$= \sum_i \{ \alpha_i \frac{\{ (\frac{P_iY_i}{PY})^{\frac{\rho}{\rho-1}} (\frac{\alpha_i}{\bar{\alpha}} \frac{K}{K_i})^{\alpha_i} (\frac{1-\alpha_i}{1-\bar{\alpha}} \frac{L}{L_i})^{1-\alpha_i} \}^{\rho-1}}{\sum_j \{ (\frac{P_jY_j}{PY})^{\frac{\rho}{\rho-1}} (\frac{\alpha_j}{\bar{\alpha}} \frac{K}{K_j})^{\alpha_j} [\frac{(1-\alpha_j)}{(1-\bar{\alpha})} \frac{L}{L_j}]^{1-\alpha_j} \}^{\rho-1}} \}, \tag{25}$$

which is clear that $\bar{\alpha}$ only depends on the expenditure share $\frac{P_jY_j}{PY}$ in the data, capital allocation $\frac{K_j}{K}$ and labor allocation $\frac{L_j}{L}$ in the data. Note that $\bar{\alpha}$ is unit-less.

In addition, one can easily verify that $\bar{\alpha} = \sum_i \frac{P_iY_i}{PY} \alpha_i$ and the following allocation of capital and labor

$$\frac{K_i}{K} = \frac{\frac{P_iY_i}{PY} \alpha_i}{\sum_i \frac{P_iY_i}{PY} \alpha_i} = \frac{\frac{P_iY_i}{PY} \alpha_i}{\bar{\alpha}} \quad \textit{and} \quad \frac{L_i}{L} = \frac{\frac{P_iY_i}{PY}(1-\alpha_i)}{\sum_i \frac{P_iY_i}{PY}(1-\alpha_i)} = \frac{\frac{P_iY_i}{PY}(1-\alpha_i)}{1-\bar{\alpha}}$$

solve equations 9 and 25 and therefore are the optimal allocation. We denote $\alpha^* = \sum_i \frac{P_iY_i}{PY} \alpha_i$,

$\chi_i^{k*} = \frac{\frac{P_i Y_i}{PY}\alpha_i}{\sum_i \frac{P_i Y_i}{PY}\alpha_i}$, and $\chi^{l*} = \frac{\frac{P_i Y_i}{PY}(1-\alpha_i)}{\sum_i \frac{P_i Y_i}{PY}(1-\alpha_i)}$. Note that the optimal allocation of capital and labor does not depend on the elasticity of substitution ρ. We can rewrite the allocative efficiency as

$$\begin{aligned}
\mathbf{E} &= \{\sum_j \{(\frac{P_j Y_j}{PY})^{\frac{\rho}{\rho-1}}(\frac{\alpha_j/K_j}{\bar{\alpha}/K})^{\alpha_j}[\frac{(1-\alpha_j)/L_j}{(1-\bar{\alpha})/L}]^{1-\alpha_j}\}^{\rho-1}\}^{\frac{1}{1-\rho}} \\
&= \{\sum_j \{(\frac{P_j Y_j}{PY})^{\frac{\rho}{\rho-1}}(\frac{\alpha_j/K_j}{\bar{\alpha}/K})^{\alpha_j}[\frac{(1-\alpha_j)/L_j}{(1-\bar{\alpha})/L}]^{1-\alpha_j}\}^{\rho-1}\}^{\frac{1}{1-\rho}}
\end{aligned}$$